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Matter coupled $F(4)$ gauged supergravity lagrangian

Laura Andrianopoli, Riccardo D'Auria and Silvia Vaulà

Dipartimento di Fisica, Politecnico di Torino
Corso Duca degli Abruzzi 24, I-10129 Torino, and
Istituto Nazionale di Fisica Nucleare (INFN)
Sezione di Torino, Italy
E-mail: andrianopol@polito.it, riccardo.d'auria@cern.ch, vaula@polito.it

Abstract: We construct the so far unknown lagrangian of $D=6$, $N=2$ $F(4)$ Supergravity coupled to an arbitrary number of vector multiplets whose scalars span the coset manifold $\frac{\mathrm{SO}(4,n)}{\mathrm{SO}(4)\times\mathrm{SO}(n)}$. This is done first in the ungauged case and then extended to the compact gauging of $\mathrm{SU}(2)\times\mathcal{G}$, where $\mathrm{SU}(2)$ is the R-symmetry diagonal subgroup of $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\simeq\mathrm{SO}(4)$ and $\mathcal{G}$ is a compact subgroup of $\mathrm{SO}(n)$, n being the number of vector multiplets, and such that $\dim(\mathcal{G})=n$. The knowledge of the lagrangian allows in principle to refine the AdS_6/CFT_5 correspondence already discussed, as far as supersymmetric multiplets are concerned, in a previous related paper. With respect to the latter we also give a more exaustive treatment of the construction of the theory at the level of superspace Bianchi identities and in particular of the scalar potential.

Keywords: AdS/CFT Correspondance, Supergravity Models.

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1. Introduction

In the classical papers of references [1, 2] it was shown that, in the classification of Lie superalgebras, there appear a few exceptional superalgebras in analogous way to what happens in the Cartan classification of Lie algebras. Among these we find a $F(4)$ superalgebra whose name is due to the fact that its construction can be realized starting from the $F(4)$ Lie algebra with a standard procedure. This superalgebra is in fact the minimal extension in $D = 5$ of the conformal group $\mathrm{SO}(2,5)$ or equivalently of the Anti de Sitter (AdS) group in six dimensions.[1] As we will see in the following, the $F(4)$ superalgebra contains as maximal bosonic subalgebra $\mathrm{so}(2,5) \otimes \mathrm{su}(2)$, so it is the natural candidate for the construction of a supergravity theory in six dimensions with 16 supersymmetries ($N = 2$, $D = 6$ supergravity). This theory was indeed constructed, without coupling to matter fields, in reference [4] and, as it was to be expected, it exhibits an AdS_6 supersymmetric background when a particular relation

[1] There exist a non-minimal supersymmetric extension of the same group giving the orthosymplectic group $\mathrm{Osp}(8^*|2)$ [3]

between the AdS_6 radius R and the coupling constant g of the gauged R-symmetry SU(2) occurs. This result was in fact retrieved after the lagrangian and the transformation rules were constructed and the extremum of the scalar potential, depending only on the dilaton field, was found. It was then realized *a posteriori* that the SU(2) gauged $N = 2$, $D = 6$ supergravity should be the looked for $F(4)$ supergravity.

In reference [5] we extended the construction to the coupling with vector multiplets (the only kind of supermatter in $D = 6$, $N = 2$ supergravity) in view of the analysis of the AdS/CFT correspondence between $F(4)$ matter coupled supergravity and the 5-dimensional superconformal field theory at the fixed point of the renormalization group, where the latter theory turns out to be a theory of interacting 5-dimensional hypermultiplets [6, 7, 8]. Our starting point, however, was different from Romans approach in that we constructed the theory directly from the $F(4)$ superalgebra or better from its dual formulation in terms of Maurer-Cartan equations (M.C.E.) suitably extended to a Free Differential Algebra (F.D.A.). In particular in this approach the relation between the SU(2) gauge coupling constant and the AdS radius appears as a natural consequence of the $F(4)$ superalgebra structure constants. Furthermore, when the M.C.E. are extended to a F.D.A. in order to include all the fields of the supergravity supermultiplet, the dynamical Higgs mechanism found by Romans, through which the antisymmetric tensor field becomes massive by eating the SU(2) gauge field singlet, turns out to be an obvious consequence of the structure of the F.D.A.

Since in reference [5] we were mainly interested in the construction of the AdS/CFT correspondence, the focus of that paper was concentrated on the construction of the supersymmetry transformation rules by solving Bianchi identities in superspace and in the subsequent construction of the scalar potential for the matter coupled theory.

In this paper we want to complete the construction of the theory by determining the matter coupled lagrangian (up to 4 fermions terms) and to give some more details on the geometrical construction of the matter coupled gauge theory and on many results which were only outlined in reference [5].

The plan of the paper is as follows: in section 2 we give the algebraic backgrounds of the $F(4)$ superalgebra together with M.C.E.'s and the extended F.D.A., explaining how in this framework the previously described results of Higgs mechanism and the g versus R_{AdS} relation arise. In section 3 we construct the matter coupled theory in absence of gauging. We do that starting from the definition of the superspace "curvatures" and solving the related Bianchi identities for the matter coupled ungauged theory. The superspace Bianchi identities solutions are then translated as ordinary transformation laws of the physical fields on space time. The ungauged lagrangian is then constructed by using the geometrical approach (rheonomic) in supespace and then restricting its form to ordinary space-time. In section 4 we perform the gauging procedure and explain how the transformation laws of the physical fields are modified both by the presence of the gauging and by turning on a mass parameter m for the antisymmetric tensor field. This is sufficient to conclude for the existence of an Anti de-Sitter supersymmetric background when $g = 3m \equiv 3(2R_{\rm AdS})^{-1}$. In section 5 the space-time lagrangian of the gauged theory is given and its properties discussed. In particular, we compute the scalar potential and verify the AdS_6/CFT_5 correspondence as far as the masses of the vector and gravitational multiplets are concerned. Section 6 contains our conclusions. The three appendices A, B and C contain technical material which is however quite essential for achieving our results. In particular, in appendices A and B we explain in some detail the superspace approach to the solution of Bianchi identities and to the construction of the lagrangian using the so called "rheonomic formalism". Note that while the rheonomic approach is fully equivalent to the ordinary superspace approach at the level of Bianchi identities, the construction of the lagrangian in the rheonomic approach has no parallel in the ordinary superspace formalism. Appendix C contains the relevant Fierz identities for the construction of the theory.

2. Geometrical and physical backgrounds

In this section we introduce the geometrical and physical settings for the construction of the theory.

Let us recall the content of the $D = 6$, $N = (1,1)$ supergravity multiplet in a Poincaré background:

$$\left(V^a_\mu, A^\alpha_\mu, B_{\mu\nu}, \psi^A_\mu, \psi^{\dot A}_\mu, \chi^A, \chi^{\dot A}, e^\sigma\right), \tag{2.1}$$

where V^a_μ is the 6-dimensional vielbein, ψ^A_μ, $\psi^{\dot A}_\mu$ are left-handed and right-handed four- component gravitino fields, respectively, A and $\dot A$ transforming under the two factors of the R-symmetry group $\mathrm{SO}(4) \simeq \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$; $B_{\mu\nu}$ is a 2-form, A^α_μ ($\alpha = 0,1,2,3$), are vector fields, $\chi^A, \chi^{\dot A}$ are left-handed and right-handed spin 1/2 four components dilatinos, and e^σ denotes the dilaton.

Our notations are as follows: $a, b, \ldots = 0,1,2,3,4,5$ are Lorentz flat indices in $D = 6$, $\mu, \nu, \ldots = 0,1,2,3,4,5$ are the corresponding world indices, $A, \dot A = 1,2$. Moreover our metric is $(+,-,-,-,-,-)$.

We recall that the description of the spinors of the multiplet in terms of left-handed and right-handed projection holds only in a Poincaré background, while in an AdS background the chiral projection cannot be defined and we are bounded to use 8-dimensional pseudo-Majorana spinors. Indeed for $\mathrm{SO}(1,5)$ (which corresponds to $D = 6$, $\rho = 4$, ρ being the signature of $\mathrm{SO}(5,1) \bmod 8$, in the notations of reference [3]), the spinors are 4-dimensional Weyl-quaternionic, while for $\mathrm{SO}(2,5)$ (corresponding to $D = 7$, $\rho = -3$), the spinors are 8-dimensional real-quaternionic.[2]

[2]Here, by "quaternionic" we mean that they satisfy a pseudo-Majorana condition.

In the former case the R-symmetry group is $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$, while in the latter case it reduces to the SU(2) diagonal subgroup of $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$. For our purposes, it is convenient to use from the very beginning 8-dimensional pseudo-Majorana spinors even in a Poincaré framework, since we are going to discuss in a unique setting both Poincaré and AdS vacua.

The pseudo-Majorana condition on the gravitino 1-forms is as follows:

$$(\psi_A)^\dagger \gamma^0 = \overline{(\psi_A)} = \epsilon^{AB} {\psi_B}^t \,, \tag{2.2}$$

where we have chosen the charge conjugation matrix in six dimensions as the identity matrix (an analogous definition holds for the dilatino fields). We use 8-dimensional antisymmetric gamma matrices, with $\gamma^7 = i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5$, which implies $\gamma_7^T = -\gamma_7$ and $(\gamma_7)^2 = -1$. The indices $A, B, \ldots = 1, 2, \ldots$ of the spinor fields ψ_A, χ_A transform in the fundamental of the diagonal subgroup SU(2) of $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$. For a generic SU(2) tensor T, raising and lowering of indices are defined by

$$\begin{aligned} T^{\ldots A \ldots} &= \epsilon^{AB} \; {T^{\ldots}}_B{}^{\ldots} \,, \\ T_{\ldots A \ldots} &= {T_{\ldots}}^B{}_{\ldots} \; \epsilon_{BA} \,. \end{aligned} \tag{2.3}$$

Taking into account that the $F(4)$ supergroup has as bosonic subgroup $\mathrm{SO}(2,5) \otimes \mathrm{SU}(2)$ we consider the 1-forms associated to AdS_6 algebra, namely ω^{ab}, V^a dual to the SO(2,5) generators M_{ab} and P_a, respectively, which satisfy:

$$\begin{aligned} [M_{ab}, M_{cd}] &= \frac{1}{2} \left(\eta_{bc} M_{ad} + \eta_{ad} M_{bc} - \eta_{bd} M_{ac} - \eta_{ac} M_{bd} \right) , \\ [P_a, P_b] &= 8m^2 M_{ab} \,, \\ [M_{ab}, P_c] &= \frac{1}{2} \left(\eta_{ac} P_b - \eta_{bc} P_a \right) \end{aligned} \tag{2.4}$$

and the 1-form A^r, $r = 1, 2, 3$ dual to the SU(2) generators T_r, satisfying:

$$[T^s; T^t] = ig\epsilon^{str} T_r \,, \tag{2.5}$$

where g is the coupling constant of SU(2), and m is related to the AdS_6 radius of $\mathrm{SO}(2,5)/\mathrm{SO}(1,5)$ by $m = (2R_{\mathrm{AdS}})^{-1}$.

In order to construct the full $F(4)$ superalgebra we now introduce the pseudo-Majorana spinor charges $Q_{A\alpha}$, (α being an 8-dimensional spinor index) and try to enlarge the $\mathrm{SO}(2,5) \otimes \mathrm{SU}(2)$ algebra to the full $F(4)$ superalgebra. The simplest procedure is to enlarge the M.C.E. of $\mathrm{SO}(2,5) \otimes \mathrm{SU}(2)$ given by:

$$\begin{aligned} \mathcal{D}V^a \equiv dV^a - \omega^{ab} V_b &= 0 \,, \\ \mathcal{R}^{ab} + 4m^2 \, V^a V^b &= 0 \,, \\ dA^r + \frac{1}{2} g \epsilon^{rst} A_s A_t &= 0 \,, \end{aligned} \tag{2.6}$$

where $\mathcal{R}^{ab} \equiv d\omega^{ab} - \omega^{ac} \wedge \; {\omega_c}^b$, in terms of the the spinor 1-forms $\psi^{A\alpha}$ dual to the odd generators $Q_{A\alpha}$. It turns out that the minimal extension of (2.6) is given by:

$$\begin{aligned}
\mathcal{D}V^a - \frac{i}{2}\,\overline{\psi}_A\gamma_a\psi^A &= 0\,,\\
\mathcal{R}^{ab} + 4m^2\,V^aV^b + m\overline{\psi}_A\gamma_{ab}\psi^A &= 0\,,\\
dA^r + \frac{1}{2}\,g\,\epsilon^{rst}A_sA_t - i\,\overline{\psi}_A\psi_B\,\sigma^{rAB} &= 0\,,\\
D\psi_A - im\gamma_a\psi_AV^a &= 0\,,
\end{aligned}\tag{2.7}$$

where D is the SO$(1,5)\otimes$SU(2) covariant derivative, which on spinors acts as follows:

$$D\psi_A \equiv \mathcal{D}\psi_A - \frac{i}{2}\sigma^r_{AB}A_r\psi^B = d\psi_A - \frac{1}{4}\gamma_{ab}\omega^{ab}\psi_A - \frac{i}{2}\sigma^r_{AB}A_r\psi^B\,. \tag{2.8}$$

Moreover $\sigma^{rAB} = \epsilon^{BC}\sigma^{rA}{}_C$, where $\sigma^{rA}{}_B$ $(r = 1,2,3)$ denote the usual Pauli matrices, are symmetric in A, B.

Note that eqs. (2.7) are closed under d-differentiation if and only if $g = 3m$. To recover this result one has to use the following Fierz identity involving 3-ψ_A's 1-forms:

$$\frac{1}{4}\gamma_{ab}\psi_A\overline{\psi}_B\gamma^{ab}\psi_C\epsilon^{AC} - \frac{1}{2}\gamma_a\psi_A\overline{\psi}_B\gamma^a\psi_C\epsilon^{AC} + 3\psi_C\overline{\psi}_B\psi_A\epsilon^{BC} = 0\,. \tag{2.9}$$

This identity is just one example of the many Fierz identities necessary for the subsequent construction of the theory. We give an account of their derivation in appendix C. At this point the Lie Algebra (anti) commutators of the $F(4)$ supergroup are easily retrieved using the well-known identity

$$d\omega(X,Y) = \frac{1}{2}\{X(\omega(Y)) - Y(\omega(X)) - \omega[X,Y]\} \tag{2.10}$$

and the duality relations given by

$$\omega^{ab}(M_{cd}) = \delta^{ab}_{cd}\,V^a(P_b) = \delta^a_b\,\psi^{A\alpha}(Q_{B\beta}) = \delta^a_b\delta^\alpha_\beta \tag{2.11}$$

all the other duality relations being zero.

The resulting $F(4)$ Lie superalgebra is:

$$\begin{aligned}
[M_{ab},M_{cd}] &= \frac{1}{2}(\eta_{bc}M_{ad} + \eta_{ad}M_{bc} - \eta_{bd}M_{ac} - \eta_{ac}M_{bd})\,,\\
[P_a,P_b] &= 8m^2M_{ab}\,,\\
[M_{ab},P_c] &= \frac{1}{2}(\eta_{ac}P_b - \eta_{bc}P_a)\,,\\
[T^s;T^t] &= ig\epsilon^{\mathrm{str}}T_r\,,\\
[M_{ab},\overline{Q}_{A\beta}] &= -\frac{1}{4}\overline{Q}_{A\alpha}(\gamma_{ab})_{\alpha\beta}\,,\\
[P_a,\overline{Q}_{A\beta}] &= im\overline{Q}_{A\alpha}(\gamma_a)_{\alpha\beta}\,,\\
[T_{(AB)},\overline{Q}_{C\alpha}] &= \frac{i}{2}g(\overline{Q}_{A\alpha}\delta_{BC} + \overline{Q}_{B\alpha}\delta_{AC})\,,\\
\{\overline{Q}_{A\alpha},Q_{B\beta}\} &= -i\epsilon_{AB}(\gamma^a)_{\alpha\beta}P_a + 4i(1)_{\alpha\beta}T_{(AB)} + m\epsilon_{AB}(\gamma^{ab})_{\alpha\beta}M_{ab}\,,
\end{aligned}\tag{2.12}$$

where we have defined $T_{(AB)} \equiv T_r\sigma^r_{AB}$ and where $g = 3m$ in this Lie-superalgebra setting is now an outcome of (super) Jacobi identities.

Let us now come back to the M.C.E.'s (2.7): note that they keep exactly the same form if we pass from the $F(4)$ supergroup to the quotient $F(4)/\,\mathrm{SO}(1,5)\otimes\mathrm{SU}(2)$, which is the relevant superspace having as bosonic subcoset AdS_6. Once the pull-back is done, the 1-forms $V^a, \omega^{ab}, \psi_A, A^r$ become superfield 1-forms whose physical meaning is given by the vielbein V^a, the spin connection ω^{ab} and the gravitino ψ_A; eqs. (2.7) then describe the vacuum configuration in superspace whose bosonic subspace is AdS_6. On the ordinary space-time, that is setting $\theta=0$ in the superfields 1-forms, the background vacuum fields have as dx^μ components the following expressions:

$$V^a_\mu = \delta^a_\mu\,; \qquad \psi_{A\mu}=0\,; \qquad \left(\omega^{ab}_\mu, A^r_\mu\right) = \text{pure gauge}\,. \tag{2.13}$$

At this point, however, it is clear that eqs. (2.7) do not describe the supersymmetric vacuum of the full $F(4)$ supergravity theory, because of the absence of the 2-form B and of the 1-form A^0 superfields, whose space-time restriction coincides with the physical fields $B_{\mu\nu}$ and A^0_μ appearing in the supergravity multiplet. The recipe to have all the fields in a single algebra is well known and consists in considering the Free Differential Algebra (F.D.A.) [9] obtained from the $F(4)$ M.C.E.'s by adding two more equations for the 2-form B and for the 1-form A^0 (the 0-form fields χ_A and σ do not appear in the algebra since they are set equal to zero[3] in the vacuum). Using the tools explained in reference [9] to construct F.D.A. containing forms of higher degree (a 2-form in our case), it turns out that the only consistent F.D.A. involving B and A^0 is given by the following extension of the (2.7):

$$\begin{aligned}
\mathcal{D}V^a - \frac{i}{2}\overline{\psi}_A\gamma_a\psi^A &= 0\,, && (2.14)\\
\mathcal{R}^{ab} + 4m^2V^aV^b + m\overline{\psi}_A\gamma_{ab}\psi^A &= 0\,, && (2.15)\\
dA^r + \frac{1}{2}\,g\,\epsilon^{rst}A_sA_t - i\,\overline{\psi}_A\psi_B\,\sigma^{rAB} &= 0\,, && (2.16)\\
dA - mB - i\,\overline{\psi}_A\gamma_7\psi^A &= 0\,, && (2.17)\\
dB + 2\,\overline{\psi}_A\gamma_7\gamma_a\psi^AV^a &= 0\,, && (2.18)\\
D\psi_A - im\gamma_a\psi_AV^a &= 0\,. && (2.19)
\end{aligned}$$

Equations (2.17) and (2.18) were obtained by imposing that together with the other Maurer Cartan equations they satisfy the d-closure requirement. Actually the closure of (2.18) relies on the 4-ψ_A's Fierz identity (see appendix C)

$$\overline{\psi}_A\gamma_7\gamma_a\psi_B\epsilon^{AB}\overline{\psi}_C\gamma^a\psi_D\epsilon^{CD} = 0\,. \tag{2.20}$$

The physical interesting feature of the F.D.A. (2.14)–(2.19) describing the full supersymmetric vacuum configuration is the appearance of the combination $dA^0 - mB$ in (2.17). When we go to the dynamical theory obtained by gauging the F.D.A. out

[3] Actually, the dilaton has to be set equal to a constant value; however, by suitable redefinition we can set the constant equal to zero.

of the vacuum, the fields A^0_μ and $B_{\mu\nu}$ will always appear in the same combination $\partial_{[\mu}A^0_{\nu]} - mB_{\mu\nu}$. At the dynamical level this implies, as noted by Romans [4], an Higgs phenomenon where the 2-form B "eats" the 1-form A^0 and acquires a non-vanishing mass m.

In summary, we have shown that two of the main results of [4], namely the existence of an AdS supersymmetric background only for $g = 3m$ and the Higgs-type mechanism by which the field $B_{\mu\nu}$ becomes massive acquiring longitudinal degrees of freedom in terms of the vector A^0_μ, are a simple consequence of the algebraic structure of the F.D.A. associated to the $F(4)$ supergroup written in terms of the vacuum-superfields.

It is interesting to see what happens if one or both the parameters g and m are zero. Setting $m = g = 0$, one reduces the $F(4)$ superalgebra to the $D = 6$, $N = (1, 1)$ superalgebra existing only in a super-Poincaré background; in this case the four vector $A^\alpha \equiv (A^0, A^r)$ transform in the fundamental of the R-symmetry group SO(4) while the pseudo-Majorana spinors ψ_A, χ_A can be decomposed in two chiral spinors, as explained at the beginning of the section, in such a way that all the resulting F.D.A. is invariant under SO(4).

Furthermore it is easy to see that no F.D.A. exists if either $m = 0$, $g \neq 0$ or $m \neq 0$, $g = 0$, since the corresponding equations in the F.D.A. do not close anymore under d- differentiation. In other words, for a supersymmetric vacuum to exist, the gauging of SU(2), $g \neq 0$, must be necessarily accompanied by the presence of the parameter m which, as we have seen, makes the closure of the supersymmetric algebra consistent for $g = 3m$.

3. The ungauged theory

In the previous section, we have fully discussed the vacuum structure of the $F(4)$ supergravity which, as we have shown, naturally admits an AdS background when $g = 3m$. Our next task is to discuss the theory out of the vacuum, that is to define appropriate curvatures for all the physical fields and to retrieve the supersymmetry transformation laws and the lagrangian of the dynamical theory. The proper way to define field strengths in the geometrical setting of superspace is to introduce "curvatures" defined as the deviation of the M.C.E. from zero, once the physical 1-forms are out of the vacuum.

However, as it happens in all supergravity theories, the dynamical theory involves the presence of a non-compact symmetry (U-duality). In our case the presence of the dilaton, which is set equal to zero in the vacuum, introduces a non-compact $O(1, 1)$ symmetry, under which the four vectors A^0, A^r transform non trivially. By suitable normalization of the dilaton field we define the action of $O(1, 1)$ as follows

$$(A^0, A^r) \longrightarrow e^\sigma (A^0, A^r) . \tag{3.1}$$

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The 2-form B is also charged under $O(1,1)$ transforming as

$$B \longrightarrow e^{-2\sigma} B \,. \tag{3.2}$$

This observation implies that when we are out of the vacuum the two parameters g and m must scale under $O(1,1)$ as follows

$$g \longrightarrow e^{-\sigma} g \,; \qquad m \longrightarrow e^{3\sigma} m \tag{3.3}$$

as it is evident from eqs. (2.16) and (2.17). Following this prescription, the gravitino curvature out of the vacuum would be defined as:

$$\rho_A = D\psi_A - ime^{-3\sigma}\gamma_a\psi_A V^a \,. \tag{3.4}$$

Note that the r.h.s. of (3.4) has no group-theoretical meaning in this case, because only in the vacuum, where $\sigma = 0$, it defines an AdS covariant derivative. For this reason, we try to build the theory starting from a Poincaré invariant vacuum which is described by the (2.7) with $g = m = 0$. Therefore we define the following set of curvatures for the Poincaré theory:

$$\begin{aligned}
T^a &= \mathcal{D}V^a - \frac{i}{2}\overline{\psi}_A\gamma_a\psi^A V^a = 0 \,, \\
R^{ab} &= \mathcal{R}^{ab} \,, \\
H &= dB + 2e^{-2\sigma}\overline{\psi}_A\gamma_7\gamma_a\psi^A V^a \,, \\
F &= dA - ie^{\sigma}\overline{\psi}_A\gamma_7\psi^A \,, \\
F^r &= dA^r - ie^{\sigma}\overline{\psi}_A\psi_B\sigma^{rAB} \,, \\
\rho_A &= D\psi_A \,, \\
R(\chi_A) &\equiv D\chi_A \,, \\
R(\sigma) &\equiv d\sigma \,.
\end{aligned} \tag{3.5}$$

In reference [5], starting from the super Poincaré curvatures and the SU(2) R-symmetry, we reproduced the results of Romans [4] for pure supergravity theory. The coupling to an arbitrary number of non-abelian gauge matter multiplets was worked out at the level of Bianchi identities. In this paper we follow the more logical step of performing the coupling to an arbitrary number of vector multiplets without gauging neither the R-symmetry nor the vector multiplets. After the theory and its lagrangian have been constructed we turn to the problem of the gauging and add the necessary new terms, proportional to the gauge coupling constants, to the lagrangian and to the transformation laws.

Let us therefore first discuss the matter vector multiplets of the theory; this is the only kind of supersymmetric matter in $D = 6$, $N = 2$. The vector multiplet is given by:

$$(A_\mu,\, \lambda_A,\, \phi^\alpha)^I \,, \tag{3.6}$$

where $\alpha = 0,1,2,3$ and the index I labels an arbitrary number n of such multiplets. As it is well known the $4n$ scalars parametrize the coset manifold $\mathrm{SO}(4,n)/\mathrm{SO}(4) \times \mathrm{SO}(n)$. Taking into account that the pure supergravity has a non-compact duality group $O(1,1)$ parametrized by e^{σ}, the duality group of the matter coupled theory is

$$G = \mathrm{SO}(4,n) \times O(1,1)\,. \tag{3.7}$$

To perform the matter coupling we follow the geometrical procedure of introducing the coset representative $L^{\Lambda}{}_{\Sigma}$ of the matter coset manifold, where $\Lambda, \Sigma, \ldots = 0, \ldots, 3+n$; decomposing the $\mathrm{SO}(4,n)$ indices with respect to $H = \mathrm{SO}(4) \times \mathrm{SO}(n)$ we have:

$$L^{\Lambda}{}_{\Sigma} = (L^{\Lambda}{}_{\alpha}, L^{\Lambda}{}_{I})\,, \tag{3.8}$$

where $\alpha = 0,1,2,3$ and $I = 4, \ldots, 3+n$. Furthermore, since we are going to gauge the $\mathrm{SU}(2)$ diagonal subgroup of $\mathrm{SO}(4)$ as in pure supergravity, we will also decompose $L^{\Lambda}{}_{\alpha}$ as

$$L^{\Lambda}{}_{\alpha} = (L^{\Lambda}{}_{0}, L^{\Lambda}{}_{r})\,, \qquad \text{with } r = 1,2,3\,. \tag{3.9}$$

The $4+n$ gravitational and matter vectors transform in the fundamental of $\mathrm{SO}(4,n)$ so that the superspace curvatures of the vectors will be now labeled by the index Λ. Furthermore the covariant derivatives acting on the spinor fields will now contain also the composite connections of the $\mathrm{SO}(4,n)$ duality group.

Let us introduce the left-invariant 1-form of $\mathrm{SO}(4,n)$

$$\Omega^{\Lambda}{}_{\Sigma} = (L^{\Lambda}{}_{\Pi})^{-1} dL^{\Pi}{}_{\Sigma} \tag{3.10}$$

satisfying the Maurer-Cartan equation

$$d\Omega^{\Lambda}{}_{\Sigma} + \Omega^{\Lambda}{}_{\Pi} \wedge \Omega^{\Pi}{}_{\Sigma} = 0\,. \tag{3.11}$$

By appropriate decomposition of the indices, we find:

$$\begin{aligned}
R^{r}{}_{s} &= -P^{r}{}_{I} \wedge P^{I}{}_{s}\,,\\
R^{r}{}_{0} &= -P^{r}{}_{I} \wedge P^{I}{}_{0}\,,\\
R^{I}{}_{J} &= -P^{I}{}_{r} \wedge P^{r}{}_{J} - P^{I}{}_{0} \wedge P^{0}{}_{J}\,,\\
\nabla P^{I}{}_{r} &= 0\,,\\
\nabla P^{I}{}_{0} &= 0\,,
\end{aligned} \tag{3.12}$$

where

$$\begin{aligned}
R^{rs} &\equiv d\Omega^{r}{}_{s} + \Omega^{r}{}_{t} \wedge \Omega^{t}{}_{s} + \Omega^{r}{}_{0} \wedge \Omega^{0}{}_{s}\,,\\
R^{r0} &\equiv d\Omega^{r}{}_{0} + \Omega^{r}{}_{t} \wedge \Omega^{t}{}_{0}\,,\\
R^{IJ} &\equiv d\Omega^{I}{}_{J} + \Omega^{I}{}_{K} \wedge \Omega^{K}{}_{J}
\end{aligned} \tag{3.13}$$

and we have set

$$P^I_\alpha = \begin{cases} P^I{}_0 \equiv \Omega^I{}_0\,, \\ P^I{}_r \equiv \Omega^I{}_r\,. \end{cases} \tag{3.14}$$

Note that P^I_0, P^I_r are the vielbeins of the coset, while $(\Omega^{rs}, \Omega^{r0})$, (R^{rs}, R^{ro}) are, respectively the connections and the curvatures of SO(4) decomposed with respect to the diagonal subgroup SU(2) $\subset$ SO(4).

In terms of the previous definitions, the ungauged superspace curvatures of the matter coupled theory are now modified, with respect to eqs. (3.5), in two aspects: first, in the definition of the superspace vector field strengths there appear, besides the $O(1,1)$ representative e^σ, also the coset representatives of the G/H σ-model, which intertwine between the R-symmetry indices A, $B, \dots$ of the gravitinos and the indices Λ, $\Sigma, \dots$ of the $4+n$-dimensional G representation; secondly, the definitions of the fermion curvatures are modified by the presence of the SU(2) connection acting on gravitinos and dilatinos, and the SU(2) and SO(n) connection on the gauginos. Therefore the ungauged superspace curvatures of the matter coupled theory are now given by:

$$\begin{aligned}
T^A &= \mathcal{D}V^a - \frac{i}{2}\,\overline{\psi}_A\gamma_a\psi^A V^a = 0\,,\\
R^{ab} &= \mathcal{R}^{ab}\,,\\
H &= dB + 2e^{-2\sigma}\,\overline{\psi}_A\gamma_7\gamma_a\psi^A V^a\,,\\
F^\Lambda &= \mathcal{F}^\Lambda - ie^\sigma L^\Lambda_0\epsilon^{AB}\overline{\psi}_A\gamma_7\psi_B - ie^\sigma L^\Lambda_r\sigma^{rAB}\overline{\psi}_A\psi_B\,,\\
\rho_A &= \mathcal{D}\psi_A - \frac{i}{2}\sigma_{rAB}\left(-\frac{1}{2}\epsilon^{rst}\Omega_{st} - i\gamma_7\Omega_{r0}\right)\psi^B\,,\\
D\chi_A &= \mathcal{D}\chi_A - \frac{i}{2}\sigma_{rAB}\left(-\frac{1}{2}\epsilon^{rst}\Omega_{st} - i\gamma_7\Omega_{r0}\right)\chi^B\,,\\
R(\sigma) &= d\sigma\,,\\
\nabla\lambda_{IA} &= \mathcal{D}\lambda_{IA} - \frac{i}{2}\sigma_{rAB}\left(-\frac{1}{2}\epsilon^{rst}\Omega_{st} - i\gamma_7\Omega_{r0}\right)\lambda^B_I + \Omega_I{}^J\lambda_{JA}\,,\\
R^I_0(\phi) &\equiv P^I_0\,,\\
R^I_r(\phi) &\equiv P^I_r\,,
\end{aligned} \tag{3.15}$$

where the last two equations define the "curvatures" of the matter scalar fields ϕ^i as the vielbein of the coset:

$$P^I_0 \equiv P^I_{0i}d\phi^i\,, \qquad P^I_r \equiv P^I_{ri}d\phi^i\,, \tag{3.16}$$

where i runs over the $4n$ values of the coset vielbein world-components.

By straightforward computation we obtain the Bianchi identities:

$$\begin{aligned}
R^{ab}V_b - i\overline{\psi}_A\gamma^a\rho_B\epsilon^{AB} &= 0\,,\\
\mathcal{D}R^{ab} &= 0\,,
\end{aligned}$$

$$
\begin{aligned}
dH + 4e^{-2\sigma} d\sigma\, \overline{\psi}_A \gamma_7 \gamma_a \psi_B \epsilon^{AB} V^a + 4e^{-2\sigma} \overline{\psi}_A \gamma_7 \gamma_a \rho_B \epsilon^{AB} V^a &= 0\,,\\
DF^\Lambda + id\sigma e^\sigma \overline{\psi}^A \gamma_7 \psi^B L^\Lambda_{[AB]} + id\sigma e^\sigma \overline{\psi}^A \psi^B L^\Lambda_{(AB)} - &\\
- 2ie^\sigma \overline{\psi}^A \gamma_7 \rho^B L^\Lambda_{[AB]} - 2ie^\sigma \overline{\psi}^A \rho^B L^\Lambda_{(AB)} + &\\
+ ie^\sigma L^\Lambda{}_I \overline{\psi}^A \psi^B P^I_{(AB)} + ie^\sigma L^\Lambda{}_I \overline{\psi}^A \gamma_7 \psi^B P^I_{[AB]} &= 0\,,\\
D\rho_A + \frac{1}{4} R^{ab} \gamma_{ab} \psi_A - \frac{i}{2} \sigma^r_{AB} \left(\frac{1}{2} \epsilon^{rst} \mathcal{R}^{st} + i\gamma_7 \mathcal{R}_{r0} \right) \psi^B &= 0\,,\\
D^2 \chi_A + \frac{1}{4} R^{ab} \gamma_{ab} \chi_A - \frac{i}{2} \sigma^r_{AB} \left(\frac{1}{2} \epsilon^{rst} \mathcal{R}^{st} + i\gamma_7 \mathcal{R}_{r0} \right) \chi^B &= 0\,,\\
d^2 \sigma &= 0\,,\\
D^2 \lambda^I_A + \frac{1}{4} R^{ab} \gamma_{ab} \lambda^I_A - \frac{i}{2} \sigma^r_{AB} \left(\frac{1}{2} \epsilon^{rst} \mathcal{R}^{st} + i\gamma_7 \mathcal{R}_{r0} \right) \lambda^{IB} - \mathcal{R}^I_J \lambda^J_A &= 0\,,\\
DP^I_{AB} &= 0\,, \qquad (3.17)
\end{aligned}
$$

where $P^I_{AB} = P^I_0 \epsilon_{AB} + P^I_r \sigma^r_{AB}$.

The solution of the Bianchi identities is a quite non-trivial task, especially when examined in the sector involving the 3 gravitino 1-forms because, as we will show in appendices A and C, we need terms of the form $\psi\psi\chi$ in the gravitino curvature superspace parametrization in order to have a consistent solution of both the fermionic and bosonic Bianchi identities. This in turn implies a full mastering of all the Fierz identities connecting different 3-ψ expressions. In appendix A we give a short account of the various techniques used in order to solve our problem, together with the solution of Bianchi identities in superspace.

Here we limit ourselves to present the solution in terms of the space-time transformation laws of the physical fields which, as is well known, can be immediately written down once the parametrizations of the supercurvatures in superspace are found. We have:

$$
\delta V^a_\mu = -i \overline{\psi}_{A\mu} \gamma^a \varepsilon^A\,, \qquad (3.18)
$$

$$
\delta B_{\mu\nu} = 4\mathrm{i} e^{-2\sigma} \overline{\chi}_A \gamma_7 \gamma_{\mu\nu} \varepsilon^A - 4e^{-2\sigma} \overline{\varepsilon}_A \gamma_7 \gamma_{[\mu} \psi^A_{\nu]}\,, \qquad (3.19)
$$

$$
\begin{aligned}
\delta A^\Lambda_\mu = {}& 2e^\sigma \overline{\varepsilon}^A \gamma_7 \gamma_\mu \chi^B L^\Lambda_0 \epsilon_{AB} + 2e^\sigma \overline{\varepsilon}^A \gamma_\mu \chi^B L^{\Lambda r} \sigma_{rAB} - e^\sigma L_{\Lambda I} \overline{\varepsilon}^A \gamma_\mu \lambda^{IB} \epsilon_{AB} + \\
& + 2ie^\sigma L^\Lambda_0 \overline{\varepsilon}_A \gamma^7 \psi_B \epsilon^{AB} + 2ie^\sigma L^{\Lambda r} \sigma^{AB}_r \overline{\varepsilon}_A \psi_B\,, \qquad (3.20)
\end{aligned}
$$

$$
\begin{aligned}
\delta\psi_{A\mu} = {}& \mathcal{D}_\mu \varepsilon_A + \frac{1}{16} e^{-\sigma} \left[T_{[AB]\nu\lambda} \gamma_7 - T_{(AB)\nu\lambda} \right] (\gamma_\mu{}^{\nu\lambda} - 6\delta^\nu_\mu \gamma^\lambda) \varepsilon^B + \\
& + \frac{i}{32} e^{2\sigma} H_{\nu\lambda\rho} \gamma_7 (\gamma_\mu{}^{\nu\lambda\rho} - 3\delta^\nu_\mu \gamma^{\lambda\rho}) \varepsilon_A + \frac{1}{2} \varepsilon_A \overline{\chi}^C \psi_{C\mu} + \\
& + \frac{1}{2} \gamma_7 \varepsilon_A \overline{\chi}^C \gamma^7 \psi_{C\mu} - \gamma_\nu \varepsilon_A \overline{\chi}^C \gamma^\nu \psi_{C\mu} + \gamma_7 \gamma_\nu \varepsilon_A \overline{\chi}^C \gamma^7 \gamma^\nu \psi_{C\mu} - \\
& - \frac{1}{4} \gamma_{\nu\lambda} \varepsilon_A \overline{\chi}^C \gamma^{\nu\lambda} \psi_{C\mu} - \frac{1}{4} \gamma_7 \gamma_{\nu\lambda} \varepsilon_A \overline{\chi}^C \gamma^7 \gamma^{\nu\lambda} \psi_{C\mu}\,, \qquad (3.21)
\end{aligned}
$$

$$
\delta\chi_A = \frac{i}{2} \gamma^\mu \partial_\mu \sigma \varepsilon_A + \frac{i}{16} e^{-\sigma} \left[T_{[AB]\mu\nu} \gamma_7 + T_{(AB)\mu\nu} \right] \gamma^{\mu\nu} \varepsilon^B +
$$

$$+\frac{1}{32}e^{2\sigma}H_{\mu\nu\lambda}\gamma_7\gamma^{\mu\nu\lambda}\varepsilon_A\,, \tag{3.22}$$

$$\delta\sigma = \overline{\chi}_A\varepsilon^A\,, \tag{3.23}$$

$$\delta\lambda^{IA} = -iP^I_{ri}\sigma^{rAB}\partial_\mu\phi^i\gamma^\mu\varepsilon_B + iP^I_{0i}\epsilon^{AB}\partial_\mu\phi^i\gamma^7\gamma^\mu\varepsilon_B + \frac{i}{2}e^{-\sigma}T^I_{\mu\nu}\gamma^{\mu\nu}\varepsilon^A\,, \tag{3.24}$$

$$P^I_{0i}\delta\phi^i = \frac{1}{2}\overline{\lambda}^I_A\gamma_7\varepsilon^A\,, \tag{3.25}$$

$$P^I_{ri}\delta\phi^i = \frac{1}{2}\overline{\lambda}^I_A\varepsilon_B\sigma^{AB}_r\,, \tag{3.26}$$

where we have introduced the "dressed" vector field strengths

$$\begin{aligned} T_{[AB]\mu\nu} &\equiv \epsilon_{AB}L^{-1}_{0\Lambda}F^\Lambda_{\mu\nu}\,,\\ T_{(AB)\mu\nu} &\equiv \sigma^r_{AB}L^{-1}_{r\Lambda}F^\Lambda_{\mu\nu}\,,\\ T_{I\mu\nu} &\equiv L^{-1}_{I\Lambda}F^\Lambda_{\mu\nu} \end{aligned} \tag{3.27}$$

and we have omitted in the transformation laws of the fermions the three-fermions terms of the form $(\chi\chi\varepsilon)$, $(\lambda\lambda\varepsilon)$, $(\lambda\chi\varepsilon)$.

Instead, we have included the extra terms in the gravitino transformation law of the form $\psi\chi\varepsilon$ which, like all the three-fermion terms, were not computed in reference [4]. However these terms correspond to terms $\psi\psi\chi$ in the superspace curvature $D\psi_A$ which, as we discuss in appendix, are quite essential to verify the consistency of the Bianchi identities; therefore they have an important meaning for the consistence of the theory and this is the reason why we have explicitly quoted them. This is to be contrasted with other three-fermion terms of the form $\chi\chi\varepsilon$, $(\lambda\lambda\varepsilon)$, $(\lambda\chi\varepsilon)$ on space-time (that is $(\chi\chi\psi)$, $(\lambda\lambda\psi)$, $(\lambda\chi\psi)$ in superspace), which we have not included in the transformation law of the fermions. Indeed their explicit form can always be found from the Bianchi identities once the consistency in the higher fermionic sectors has been verified, so that they are not on the same status. Since their explicit computation is very cumbersome they have not been computed.

An important property of the solution presented is the fact that, in the ungauged theory, no supersymmetric AdS background exists, as it was expected from the discussion given in section 2, while the Poincaré supersymmetric vacuum does exist. Indeed if we go in the vacuum, where all the field strengths of the bosonic fields and the fermionic fields are zero, and the scalar σ takes an arbitrary constant value (which we set equal to zero), one easily derives that under supersymmetry eqs. (3.21), (3.22) and (3.24) reduce to:

$$\delta\psi_{A\mu} = \mathcal{D}_\mu\varepsilon_A\,, \tag{3.28}$$

$$\delta\chi_A = 0\,, \tag{3.29}$$

$$\delta\lambda^{IA} = 0 \tag{3.30}$$

which proves our statement. In the next section, we will gauge the theory and we will see that in presence of gauging we find a more general solution of the Bianchi identities containing a new parameter m, such that in the vacuum a supersymmetric AdS_6 background can be retrieved.

We observe that the solutions of the Bianchi identities also imply the equations of motion of the physical fields and therefore one can reconstruct in principle the space-time lagrangian. Nevertheless, in general, it is simpler to construct the lagrangian explicitly. In order to do that, we use a superspace geometric approach, called "rheonomic" [9], for the construction of a lagrangian in superspace, which turns out to be greatly simplified once the Bianchi identities have been solved. Its construction is sketched in appendix B. Here we just quote the space-time lagrangian which is obtained from the rheonomic one by restricting all the fields to their space-time values (see appendix B). The final expression is the following:

$$\mathcal{A} = \int \mathcal{L}^{(D=6,N=2)}_{(\text{ungauged})} \sqrt{-g}\ d^6x \,, \tag{3.31}$$

where:

$$\mathcal{L}^{(D=6,N=2)}_{(\text{ungauged})} \equiv \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{Pauli}} + \mathcal{L}_{\text{Chern-Simons}} + \mathcal{L}_{4\,\text{fermions}} \tag{3.32}$$

and:

$$\begin{aligned}
\mathcal{L}_{\text{kin}} &= -\frac{1}{4}\mathcal{R} - \frac{1}{8}e^{2\sigma}\mathcal{N}_{\Lambda\Sigma}\mathcal{F}^{\Lambda}_{\mu\nu}\mathcal{F}^{\Sigma\mu\nu} + \frac{3}{64}e^{4\sigma}H_{\mu\nu\rho}H^{\mu\nu\rho} + \\
&\quad + \frac{i}{2}\overline{\psi}_{A\mu}\gamma^{\mu\nu\rho}D_\nu\psi^A_\rho - 2i\overline{\chi}_A\gamma^\mu D_\mu\chi^A + \frac{i}{8}\overline{\lambda}^I_A\gamma^\mu D_\mu\lambda^A_I + \\
&\quad + \partial^\mu\sigma\partial_\mu\sigma - \frac{1}{4}\left(P^{I0}_i P_{I0j} + P^{Ir}_i P_{Irj}\right)\partial^\mu\phi^i\partial_\mu\phi^j\,; \\
\mathcal{L}_{\text{Pauli}} &= -2\partial_\mu\sigma\overline{\chi}_A\gamma^\nu\gamma^\mu\psi^A_\nu + \frac{1}{4}P^{I0}_i\partial^\mu\phi^i\overline{\lambda}_{IA}\gamma_7\gamma^\nu\gamma^\mu\psi^A_\nu + \\
&\quad + \frac{1}{4}P^{I(AB)}_i\partial^\mu\phi^i\overline{\lambda}_{IA}\gamma^\nu\gamma^\mu\psi_{B\nu} + \\
&\quad + e^{-\sigma}\mathcal{N}_{\Lambda\Sigma}\mathcal{F}^\Lambda_{\mu\nu}\left\{\frac{i}{8}L^\Sigma_0\overline{\psi}^\rho_A\gamma_7\left(\gamma^{\mu\nu}{}_{\rho\sigma} + 2\delta^{\mu\nu}_{\rho\sigma}\right)\psi^{A\sigma} + \right. \\
&\qquad\qquad + \frac{i}{8}L^\Sigma_{(AB)}\overline{\psi}^{A\rho}\left(\gamma^{\mu\nu}{}_{\rho\sigma} + 2\delta^{\mu\nu}_{\rho\sigma}\right)\psi^{B\sigma} - \\
&\qquad\qquad - \frac{1}{4}L^\Sigma_0\overline{\psi}_{A\rho}\gamma_7\gamma^{\mu\nu}\gamma^\rho\chi^A - \frac{1}{4}L^\Sigma_{(AB)}\overline{\psi}^A_\rho\gamma^{\mu\nu}\gamma^\rho\chi^B - \\
&\qquad\qquad - \frac{1}{8}L^\Sigma_I\overline{\lambda}^I_A\gamma^{\mu\nu}\gamma^\rho\psi^A_\rho + \\
&\qquad\qquad + \frac{1}{2}L^\Sigma_0\left(\frac{\mathrm{i}}{2}\overline{\chi}_A\gamma_7\gamma^{\mu\nu}\chi^A + \frac{\mathrm{i}}{16}\overline{\lambda}_{IA}\gamma_7\gamma^{\mu\nu}\lambda^{IA}\right) + \\
&\qquad\qquad \left. + \frac{1}{2}L^\Sigma_{(AB)}\left(-\frac{\mathrm{i}}{2}\overline{\chi}^A\gamma^{\mu\nu}\chi^B + \frac{\mathrm{i}}{16}\overline{\lambda}^A_I\gamma^{\mu\nu}\lambda^{IB}\right)\right\} + \\
&\quad + \frac{3}{8}e^{2\sigma}H_{\mu\nu\rho}\left\{\frac{1}{2}\overline{\psi}^\lambda_A\gamma_7\left(\delta^{\mu\nu}_{\lambda\sigma}\gamma^\rho - 16\gamma^{\mu\nu\rho}{}_{\lambda\sigma}\right)\psi^{A\sigma} + \right.
\end{aligned}$$

$$+\frac{i}{3}\overline{\psi}_{A\sigma}\gamma_7\gamma^\sigma\gamma^{\mu\nu\rho}\chi^A+\frac{1}{3}\overline{\chi}_A\gamma_7\gamma^{\mu\nu\rho}\chi^A\Big\};$$

$$\mathcal{L}_{\text{Chern-Simons}}=-\frac{1}{64}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}\eta_{\Lambda\Sigma}\mathcal{F}^\Lambda_{\rho\sigma}\mathcal{F}^\Sigma_{\lambda\tau}\,, \tag{3.33}$$

where we have defined [11]:

$$\mathcal{N}_{\Lambda\Sigma}=(L^{-1})_{AB|\Lambda}(L^{-1})^{AB}{}_\Sigma-(L^{-1})_{I|\Lambda}(L^{-1})^I{}_\Sigma\,. \tag{3.34}$$

4. The gauging

The next problem we have to cope with is the gauging of the matter coupled theory and the determination of the scalar potential.

Let us first consider the ordinary gauging, with $m=0$, which, as usual, will imply the presence of new terms proportional to the coupling constants in the supersymmetry transformation laws of the fermion fields.

Our aim is to gauge a compact subgroup of $\mathrm{SO}(4,n)$. Since in any case we may gauge only the diagonal subgroup $\mathrm{SU}(2)\subset\mathrm{SO}(4)\subset\mathrm{SO}(4)\otimes\mathrm{SO}(n)$, the maximal gauging is given by $\mathrm{SU}(2)\otimes\mathcal{G}$ where $\mathcal{G}$ is a n-dimensional subgroup of $\mathrm{SO}(n)$. According to a well-known procedure, we modify the definition of the left invariant 1-form $L^{-1}dL$ by replacing the ordinary differential with the $\mathrm{SU}(2)\otimes\mathcal{G}$ covariant differential as follows:

$$\nabla L^\Lambda{}_\Sigma=dL^\Lambda{}_\Sigma-f_\Gamma{}^\Lambda{}_\Pi A^\Gamma L^\Pi{}_\Sigma\,, \tag{4.1}$$

where $f^\Lambda{}_{\Pi\Gamma}$ are the structure constants of $\mathrm{SU}(2)\otimes\mathcal{G}$, $\mathrm{SU}(2)$ being the diagonal subgroup of $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R\simeq\mathrm{SO}(4)$. More explicitly, denoting with ϵ^{rst} and $\mathcal{C}^{IJK}$ the structure constants of the two factors $\mathrm{SU}(2)$ and $\mathcal{G}$, eq. (4.1) splits as follows:

$$\begin{aligned}\nabla L^0{}_\Sigma&=dL^\Lambda{}_\Sigma\,,\\ \nabla L^r{}_\Sigma&=dL^r{}_\Sigma-g\epsilon_t{}^r{}_sA^tL^s{}_\Sigma\,,\\ \nabla L^I{}_\Sigma&=dL^I{}_\Sigma-g'\mathcal{C}_K{}^I{}_JA^KL^J{}_\Sigma\,.\end{aligned} \tag{4.2}$$

Setting $\widehat{\Omega}=L^{-1}\nabla L$, one easily obtains the gauged Maurer-Cartan equations:

$$d\widehat{\Omega}^\Lambda{}_\Sigma+\widehat{\Omega}^\Lambda{}_\Pi\wedge\widehat{\Omega}^\Pi{}_\Sigma=(L^{-1}\mathcal{F}L)^\Lambda{}_\Sigma\,, \tag{4.3}$$

where $\mathcal{F}\equiv\mathcal{F}^\Lambda T_\Lambda$, T_Λ being the generators of $\mathrm{SU}(2)\otimes\mathcal{G}$.

After gauging, the same decomposition as in eqs. (3.12) now gives:

$$\begin{aligned}\widehat{R}^r{}_s&=R^r{}_s+(L^{-1}\mathcal{F}L)^r{}_s=-P^r_I\wedge P^I_s+(L^{-1}\mathcal{F}L)^r{}_s\,,\\ \widehat{R}^r{}_0&=R^r+(L^{-1}\mathcal{F}L)^r{}_0=-P^r_I\wedge P^I_0+(L^{-1}\mathcal{F}L)^r{}_0\,,\\ \widehat{R}^I{}_J&=R^I{}_J+(L^{-1}\mathcal{F}L)^I{}_J=-P^I_r\wedge P^r_J-P^I_0\wedge P^0_J+(L^{-1}\mathcal{F}L)^I{}_J\,,\\ \nabla\widehat{P}^I{}_r&=(L^{-1}\mathcal{F}L)^I{}_r\,,\\ \nabla\widehat{P}^I{}_0&=(L^{-1}\mathcal{F}L)^I{}_0\,,\end{aligned} \tag{4.4}$$

where:

$$\widehat{P}^I{}_0 = \left(L^{-1}\right)^I{}_\Lambda \nabla L^\Lambda{}_0 \,,$$
$$\widehat{P}^I{}_r = \left(L^{-1}\right)^I{}_\Lambda \nabla L^\Lambda{}_r \,. \tag{4.5}$$

Because of the presence of the gauged terms in the coset curvatures, the new Bianchi Identities (whose explicit form is given in appendix A) are not satisfied by the old superspace curvatures. Therefore we obtain a different solution for the "curvatures" which, in space-time language, amounts to different transformation laws. However, the new solution for the curvatures and hence the new transformation laws can be obtained from the old ones by performing in (3.18)–(3.26) the following modifications:

1. The vector field-strengths $F^\Lambda_{\mu\nu}$ are now non abelian.
2. For the vielbein of the scalar manifold, we perform the replacement: $(P^I_{0i}, P^I_{ri}) \to (\widehat{P}^I_{0i}, \widehat{P}^I_{ri})$, according to eq. (4.5).
3. This is the most important modification: the transformation rules of the Fermi fields require extra terms proportional to the gauge coupling constants, called "fermionic shifts". In particular, if we denote by $\widehat{\delta\psi}^{(\mathrm{old})}_{A\mu}$, $\widehat{\delta\chi}^{(\mathrm{old})}_{A}$ and $\widehat{\delta\lambda}^{I(\mathrm{old})}_{A}$ the transformation laws (3.21), (3.22) and (3.24) modified according to items 1. and 2. we may write:

$$\delta\psi_{A\mu} = \widehat{\delta\psi}^{(\mathrm{old})}_{A\mu} + S^{(g,g')}_{AB}\gamma_\mu\varepsilon^B \,,$$
$$\delta\chi_A = \widehat{\delta\chi}^{(\mathrm{old})}_{A} + N^{(g,g')}_{AB}\varepsilon^B \,,$$
$$\delta\lambda^I_A = \widehat{\delta\lambda}^{I(\mathrm{old})}_{A} + M^{I(g,g')}_{AB}\varepsilon^B \,. \tag{4.6}$$

Working out the Bianchi identities, one fixes the explicit form of the fermionic shifts which turn out to be

$$S^{(g,g')}_{AB} = \frac{i}{24}Ae^\sigma\epsilon_{AB} - \frac{i}{8}B_t\gamma^7\sigma^t_{AB} \,,$$
$$N^{(g,g')}_{AB} = \frac{1}{24}Ae^\sigma\epsilon_{AB} + \frac{1}{8}B_t\gamma^7\sigma^t_{AB} \,,$$
$$M^{I(g,g')}_{AB} = (-C^I_t + 2i\gamma^7 D^I_t)\sigma^t_{AB} \,, \tag{4.7}$$

where

$$A = \epsilon^{rst}K_{rst} \,, \tag{4.8}$$
$$B^i = \epsilon^{ijk}K_{jk0} \,, \tag{4.9}$$
$$C^t_I = \epsilon^{trs}K_{rIs} \,, \tag{4.10}$$
$$D_{It} = K_{0It} \tag{4.11}$$

and the threefold completely antisymmetric tensors $K's$ are the so called "boosted structure constants" given explicitly by:

$$\begin{aligned}
K_{rst} &= g\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^m L^n{}_t + g'\mathcal{C}_{IJK}L^I{}_r(L^{-1})_s{}^J L^K{}_t\,,\\
K_{rs0} &= g\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^m L^n{}_0 + g'\mathcal{C}_{IJK}L^I{}_r(L^{-1})_s{}^J L^K{}_0\,,\\
K_{rIt} &= g\epsilon_{lmn}L^l{}_r(L^{-1})_I{}^m L^n{}_t + g'\mathcal{C}_{LJK}L^L{}_r(L^{-1})_I{}^J L^K{}_t\,,\\
K_{0It} &= g\epsilon_{lmn}L^l{}_0(L^{-1})_I{}^m L^n{}_t + g'\mathcal{C}_{LJK}L^L{}_0(L^{-1})_I{}^J L^K{}_t\,.
\end{aligned} \tag{4.12}$$

However, this is not the most general solution of the Bianchi identities. Indeed, even in absence of scalar matter fields, there is a more general solution of the Bianchi identities which involve a new parameter m which behaves like a second "gauging" since it only affects the transformation laws of Fermi fields with suitable shifts proportional to m in the transformation laws of the fermions, provided we also redefine the singlet vector field-strength as:

$$F \longrightarrow \widehat{F} \equiv F - mB\,. \tag{4.13}$$

These m-shifts in the pure gravitational case are given by:

$$\begin{aligned}
S^{(m)}_{AB} &= \frac{\mathrm{i}}{4}me^{-3\sigma}\gamma_7\epsilon_{AB}\,,\\
N^{(m)}_{AB} &= -\frac{3}{4}me^{-3\sigma}\epsilon_{AB}
\end{aligned} \tag{4.14}$$

which, in presence of matter multiplets, generalize to:

$$\begin{aligned}
S^{(m)}_{AB} &= \frac{\mathrm{i}}{4}me^{-3\sigma}(L^{-1})_{00}\epsilon_{AB} + \frac{i}{4}me^{-3\sigma}(L^{-1})_{i0}\gamma_7\sigma^t_{AB}\,,\\
N^{(m)}_{AB} &= -\frac{3}{4}me^{-3\sigma}(L^{-1})_{00}\epsilon_{AB} + \frac{3}{4}me^{-3\sigma}(L^{-1})_{i0}\gamma_7\sigma^t_{AB}\,,\\
M^{I(m)}_{AB} &= -2me^{-3\sigma}(L^{-1})^I{}_0\epsilon_{AB}\,.
\end{aligned} \tag{4.15}$$

Hence, we may have two different theories either gauging $\mathrm{SU}(2)\times\mathcal{G}$, with shifts given by eqs. (4.7), or performing the second gauging proportional to m with shifts given by eqs. (4.15). Neither of these two theories have an invariant supersymmetric Poincaré or Anti de Sitter background what is in agreement with the discussion of the F.D.A. given at the end of section 2. Actually, these two gaugings do not interfere and we may perform both at once obtaining in this way the following general form for the fermionic shifts:

$$\begin{aligned}
S^{(g,g',m)}_{AB} &= \frac{\mathrm{i}}{24}\left[Ae^{\sigma} + 6me^{-3\sigma}(L^{-1})_{00}\right]\epsilon_{AB} - \frac{i}{8}\left[B_te^{\sigma} - 2me^{-3\sigma}(L^{-1})_{i0}\right]\gamma^7\sigma^t_{AB}\,,\\
N^{(g,g',m)}_{AB} &= \frac{1}{24}\left[Ae^{\sigma} - 18me^{-3\sigma}(L^{-1})_{00}\right]\epsilon_{AB} + \frac{1}{8}\left[B_te^{\sigma} + 6me^{-3\sigma}(L^{-1})_{i0}\right]\gamma^7\sigma^t_{AB}\,,\\
M^{I(g,g',m)}_{AB} &= (-C^I_t + 2i\gamma^7 D^I_t)e^{\sigma}\sigma^t_{AB} - 2me^{-3\sigma}(L^{-1})^I{}_0\epsilon_{AB}\,,
\end{aligned} \tag{4.16}$$

besides the new definition $\widehat{F} \equiv F - mB$.

We thus obtain the supersymmetry transformation rules in presence of gauging and with the mass parameter turned on:

$$
\begin{aligned}
\delta V^a_\mu &= -i\overline{\psi}_{A\mu}\gamma^a\varepsilon^A\,,\\
\delta B_{\mu\nu} &= 4\mathrm{i}e^{-2\sigma}\overline{\chi}_A\gamma_7\gamma_{\mu\nu}\varepsilon^A - 4e^{-2\sigma}\overline{\varepsilon}_A\gamma_7\gamma_{[\mu}\psi^A_{\nu]}\,,\\
\delta A^\Lambda_\mu &= 2e^\sigma\overline{\varepsilon}^A\gamma_7\gamma_\mu\chi^B L^\Lambda_0\epsilon_{AB} + 2e^\sigma\overline{\varepsilon}^A\gamma_\mu\chi^B L^{\Lambda r}\sigma_{rAB} - e^\sigma L_{\Lambda I}\overline{\varepsilon}^A\gamma_\mu\lambda^{IB}\epsilon_{AB} +\\
&\quad + 2ie^\sigma L^\Lambda_0\overline{\varepsilon}_A\gamma^7\psi_B\epsilon^{AB} + 2ie^\sigma L^{\Lambda r}\sigma^{AB}_r\overline{\varepsilon}_A\psi_B\,,\\
\delta\psi_{A\mu} &= \mathcal{D}_\mu\varepsilon_A + \frac{1}{16}e^{-\sigma}\left[\widehat{T}_{[AB]\nu\lambda}\gamma_7 - T_{(AB)\nu\lambda}\right](\gamma_\mu{}^{\nu\lambda} - 6\delta^\nu_\mu\gamma^\lambda)\varepsilon^B +\\
&\quad + \frac{i}{32}e^{2\sigma}H_{\nu\lambda\rho}\gamma_7(\gamma_\mu{}^{\nu\lambda\rho} - 3\delta^\nu_\mu\gamma^{\lambda\rho})\varepsilon_A + S^{(g,g',m)}_{AB}\gamma_\mu\varepsilon^B +\\
&\quad + \frac{1}{2}\varepsilon_A\overline{\chi}^C\psi_{C\mu} + \frac{1}{2}\gamma_7\varepsilon_A\overline{\chi}^C\gamma^7\psi_{C\mu} - \gamma_\nu\varepsilon_A\overline{\chi}^C\gamma^\nu\psi_{C\mu} + \gamma_7\gamma_\nu\varepsilon_A\overline{\chi}^C\gamma^7\gamma^\nu\psi_{C\mu} -\\
&\quad - \frac{1}{4}\gamma_{\nu\lambda}\varepsilon_A\overline{\chi}^C\gamma^{\nu\lambda}\psi_{C\mu} - \frac{1}{4}\gamma_7\gamma_{\nu\lambda}\varepsilon_A\overline{\chi}^C\gamma^7\gamma^{\nu\lambda}\psi_{C\mu}\,,\\
\delta\chi_A &= \frac{i}{2}\gamma^\mu\partial_\mu\sigma\varepsilon_A + \frac{i}{16}e^{-\sigma}\left[\widehat{T}_{[AB]\mu\nu}\gamma_7 + T_{(AB)\mu\nu}\right]\gamma^{\mu\nu}\varepsilon^B + \frac{1}{32}e^{2\sigma}H_{\mu\nu\lambda}\gamma_7\gamma^{\mu\nu\lambda}\varepsilon_A +\\
&\quad + N^{(g,g',m)}_{AB}\epsilon^B\,,\\
\delta\sigma &= \overline{\chi}_A\varepsilon^A\,,\\
\delta\lambda^I_A &= i\widehat{P}^I_{ri}\sigma^r_{AB}\partial_\mu\phi^i\gamma^\mu\varepsilon^B - i\widehat{P}^I_{0i}\epsilon_{AB}\partial_\mu\phi^i\gamma^7\gamma^\mu\varepsilon^B + \frac{i}{2}e^{-\sigma}T^I_{\mu\nu}\gamma^{\mu\nu}\varepsilon_A + M^{I(g,g',m)}_{AB}\varepsilon^B\,,\\
\widehat{P}^I_{0i}\delta\phi^i &= \frac{1}{2}\overline{\lambda}^I_A\gamma_7\varepsilon^A\,, \qquad \widehat{P}^I_{ri}\delta\phi^i = \frac{1}{2}\overline{\lambda}^I_A\varepsilon_B\sigma^{AB}_r\,,
\end{aligned}
\tag{4.17}
$$

where we have introduced the "dressed" non-abelian vector field strengths:

$$
\begin{aligned}
\widehat{T}_{[AB]\mu\nu} &\equiv \epsilon_{AB}L^{-1}_{0\Lambda}\left(F^\Lambda_{\mu\nu} - mB_{\mu\nu}\delta_{\Lambda 0}\right),\\
T_{(AB)\mu\nu} &\equiv \sigma^r_{AB}L^{-1}_{r\Lambda}F^\Lambda_{\mu\nu}\,,\\
T_{I\mu\nu} &\equiv L^{-1}_{I\Lambda}F^\Lambda_{\mu\nu}
\end{aligned}
\tag{4.18}
$$

the gauged scalar vielbein $\widehat{P}^I_0$, $\widehat{P}^I_r$ being defined in eq. (4.5).

From the transformation laws (4.17), it is easy to see that one can obtain an Anti de Sitter supersymmetric background choosing $g = 3m$. (Recall that in the vacuum, besides putting to zero the field-strengths, we also set $\sigma = 0$, $L^\Lambda{}_\Sigma = \delta^\Lambda_\Sigma$). In this way we obtain:

$$
\begin{aligned}
\delta\chi_A &\equiv -\frac{1}{4}(g - 3m)\,\varepsilon_A = 0\,,\\
\delta\psi_{A\mu} &\equiv D_\mu\varepsilon_A - \mathrm{i}\left(\frac{3}{4}m + \frac{1}{4}g\right)\gamma_\mu\varepsilon_A = D_\mu\varepsilon_A - \mathrm{i}m\gamma_\mu\varepsilon_A = \nabla^{\mathrm{AdS}}_\mu\epsilon_A\,,\\
R^{ab} &\equiv -\frac{1}{2}R^{ab}_{cd}V^cV^d = -4m^2V^aV^b \longrightarrow R_{\mu\nu} = 20m^2g_{\mu\nu}\,,\\
&\qquad \left(\mathcal{F}^r_{\mu\nu} = \mathcal{F}_{\mu\nu} - mB_{\mu\nu} = H_{\mu\nu\rho} = \chi_A = \psi_{A\mu} = \sigma = 0;\ L^\Lambda{}_\Sigma = \delta^\Lambda_\Sigma\right)
\end{aligned}
\tag{4.19}
$$

which corresponds to an AdS configuration with AdS radius $R^2_{\mathrm{AdS}} = (4m^2)^{-1}$

5. The complete gauged lagrangian

When the gauging is present the lagrangian can be constructed by covariantization (with respect to the gauge group) of all the derivatives present in the ungauged one (3.32) plus extra terms, bilinears in the fermions, proportional to the coupling constants g, g' and m and a scalar potential which is quadratic in the same parameters. As for the ungauged case, the explicit construction has been done using the rheonomic formalism in superspace and taking advantage of the parametrizations of the curvatures obtained by solving the Bianchi identities. A short account of this is given in appendix B. By restricting the superspace lagrangian to space-time we get:

$$\begin{aligned}
\mathcal{A}_{(D=6,N=2)} &= \int \mathcal{L}^{(g,g',m)}_{(D=6,N=2)}\sqrt{-g}\, d^6x \,, \\
\mathcal{L}^{(g,g',m)}_{(D=6,N=2)} &\equiv \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm Pauli} + \mathcal{L}_{\texttt{Chern-Simons}} + \mathcal{L}_{\rm gauging} + \mathcal{L}_{4\,\rm fermions}\,,
\end{aligned} \tag{5.1}$$

where:

$$\begin{aligned}
\mathcal{L}_{\rm kin} &= -\frac{1}{4}\mathcal{R} - \frac{1}{8}e^{-2\sigma}\mathcal{N}_{\Lambda\Sigma}\widehat{\mathcal{F}}^{\Lambda}_{\mu\nu}\widehat{\mathcal{F}}^{\Sigma\mu\nu} + \frac{3}{64}e^{4\sigma}H_{\mu\nu\rho}H^{\mu\nu\rho} + \\
&\quad + \frac{i}{2}\overline{\psi}_{A\mu}\gamma^{\mu\nu\rho}\nabla_\nu\psi^A_\rho - 2i\overline{\chi}_A\gamma^\mu\nabla_\mu\chi^A - \frac{i}{8}\overline{\lambda}^I_A\gamma^\mu\nabla_\mu\lambda^A_I + \\
&\quad + \partial^\mu\sigma\partial_\mu\sigma - \frac{1}{4}\left(\widehat{P}^{I0}_i\widehat{P}_{I0j} + \widehat{P}^{Ir}_i\widehat{P}_{Ir,j}\right)\partial^\mu\phi^i\partial_\mu\phi^j\,; \\
\mathcal{L}_{\rm Pauli} &= -2\partial_\mu\sigma\overline{\chi}_A\gamma^\nu\gamma^\mu\psi^A_\nu + \frac{1}{4}\widehat{P}^{I0}_i\partial^\mu\phi^i\overline{\lambda}_{IA}\gamma_7\gamma^\nu\gamma^\mu\psi^A_\nu + \\
&\quad + \frac{1}{4}\widehat{P}^{I(AB)}_i\partial^\mu\phi^i\overline{\lambda}_{IA}\gamma^\nu\gamma^\mu\psi_{B\nu} + \\
&\quad + e^{-\sigma}\mathcal{N}_{\Lambda\Sigma}\widehat{\mathcal{F}}^{\Lambda}_{\mu\nu}\left\{\frac{i}{8}L^\Sigma_0\overline{\psi}^\rho_A\gamma_7\left(\gamma^{\mu\nu}{}_{\rho\sigma} + 2\delta^{\mu\nu}_{\rho\sigma}\right)\psi^{A\sigma} + \right. \\
&\qquad\qquad + \frac{i}{8}L^\Sigma_{(AB)}\overline{\psi}^{A\rho}\left(\gamma^{\mu\nu}{}_{\rho\sigma} + 2\delta^{\mu\nu}_{\rho\sigma}\right)\psi^{B\sigma} - \\
&\qquad\qquad - \frac{1}{4}L^\Sigma_0\overline{\psi}_{A\rho}\gamma_7\gamma^{\mu\nu}\gamma^\rho\chi^A - \frac{1}{4}L^\Sigma_{(AB)}\overline{\psi}^A_\rho\gamma^{\mu\nu}\gamma^\rho\chi^B - \\
&\qquad\qquad - \frac{1}{8}L^\Sigma_I\overline{\lambda}^I_A\gamma^{\mu\nu}\gamma^\rho\psi^A_\rho + \\
&\qquad\qquad + L^\Sigma_0\left(\frac{\mathrm{i}}{4}\overline{\chi}_A\gamma_7\gamma^{\mu\nu}\chi^A + \frac{\mathrm{i}}{32}\overline{\lambda}_{IA}\gamma_7\gamma^{\mu\nu}\lambda^{IA}\right) + \\
&\qquad\qquad \left. + L^\Sigma_{(AB)}\left(-\frac{\mathrm{i}}{4}\overline{\chi}^A\gamma^{\mu\nu}\chi^B + \frac{\mathrm{i}}{32}\overline{\lambda}^A_I\gamma^{\mu\nu}\lambda^{IB}\right)\right\} + \\
&\quad + \frac{3}{8}e^{2\sigma}H_{\mu\nu\rho}\left\{\frac{1}{2}\overline{\psi}^\lambda_A\gamma_7\left(\delta^{\mu\nu}_{\lambda\sigma}\gamma^\rho - 16\gamma^{\mu\nu\rho}{}_{\lambda\sigma}\right)\psi^{A\sigma} + \frac{i}{3}\overline{\psi}_{A\sigma}\gamma_7\gamma^\sigma\gamma^{\mu\nu\rho}\chi^A + \right. \\
&\qquad\qquad \left. + \frac{i1}{3}\overline{\chi}_A\gamma_7\gamma^{\mu\nu\rho}\chi^A\right\}; \\
\mathcal{L}_{\texttt{Chern-Simons}} &= -\frac{1}{64}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}\left(\eta_{\Lambda\Sigma}\widehat{\mathcal{F}}^\Lambda_{\rho\sigma}\widehat{\mathcal{F}}^\Sigma_{\lambda\tau} + mB_{\rho\sigma}\widehat{\mathcal{F}}^0_{\lambda\tau} + \frac{1}{3}m^2B_{\rho\sigma}B_{\lambda\tau}\right),
\end{aligned}$$

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$$
\begin{aligned}
\mathcal{L}_{\text{gauging}} &= 2\mathrm{i}\overline{\psi}^A_\mu \gamma^{\mu\nu} \overline{S}_{AB} \psi^B_\nu + 4\mathrm{i}\overline{\psi}^A_\mu \gamma^\mu \overline{N}_{AB} \chi^B - \frac{\mathrm{i}}{4}\overline{\psi}^A_\mu \gamma^\mu \overline{M}^I_{AB} \lambda^B_I + \\
&\quad + \overline{\chi}^A X_{AB} \chi^B + \overline{\chi}^A Y^I_{AB} \lambda^B_I + \overline{\lambda}^I_A Z^{AB}_{IJ} \lambda^J_B - \mathcal{W}(\sigma, \phi^i; g, g', m) \,. \qquad (5.2)
\end{aligned}
$$

where the covariant derivatives in $\mathcal{L}_{\text{kin}}$ are now Lorentz- and $\mathrm{SU}(2) \otimes \mathcal{G}$-covariant derivatives. $\mathcal{L}_{4\,\text{fermions}}$ has not been computed.

In eq. (5.2) there appear "barred mass-matrices" $\overline{S}_{AB}$, $\overline{N}_{AB}$, $\overline{M}^I_{AB}$ which are slightly different from the fermionic shifts defined in eqs. (4.15). Actually they are defined by:

$$
\overline{S}_{AB} = -S_{BA}\,, \qquad \overline{N}_{AB} = -N_{BA}\,, \qquad \overline{M}^I_{AB} = M^I_{BA}\,. \qquad (5.3)
$$

Definitions (5.3) stem from the fact that the shifts defined in eqs. (4.15) are matrices in the 8-dimensional spinor space, since they contain the γ_7 matrix; as will be seen in a moment, such definitions are actually necessary in order to satisfy the supersymmetry Ward identities.

Furthermore, the mass matrices of the spin one-half fermions and the potential can be computed from supersymmetry of the lagrangian and turn out to be:

$$
\begin{aligned}
X_{AB} &= 4\mathrm{i}\left(S_{AB} - 2\mathrm{i}\overline{N}_{AB}\right), \\
Y^I_{AB} &= -\frac{1}{2}\partial_\sigma \overline{M}^I_{AB}\,, \\
Z^{AB}_{IJ} &= \frac{\mathrm{i}}{4}\left[\left(\overline{S}^{AB} + \mathrm{i}N^{AB}\right)\eta_{IJ} + \left(K_{rIJ}\sigma^{rAB} - K_{0IJ}\gamma_7\epsilon^{AB}\right)\right], \qquad (5.4) \\
\mathcal{W}(\phi) &= -5\left\{\left[\frac{1}{12}(Ae^\sigma + 6me^{-3\sigma}L_{00})\right]^2 + \left[\frac{1}{4}(e^\sigma B_r - 2me^{-3\sigma}L_{0r})\right]^2\right\} + \\
&\quad + \left\{\left[\frac{1}{12}(Ae^\sigma - 18me^{-3\sigma}L_{00})\right]^2 + \left[\frac{1}{4}(e^\sigma B_r + 6me^{-3\sigma}L_{0r})\right]^2\right\} - \\
&\quad - \frac{1}{4}\{C^I{}_t C_{It} + 4D^I{}_t D_{It}\}\; e^{2\sigma} - m^2 e^{-6\sigma} L_{0I} L^{0I}\,. \qquad (5.5)
\end{aligned}
$$

It is convenient to discuss in detail the determination of the potential from the supersymmetry variation of the lagrangian. Indeed, let us perform the supersymmetry variation of (5.1), keeping only the terms quadratic in g, g' or m, proportional to the currents $\overline{\psi}_{A\mu}\gamma^\mu\epsilon^C$ and $\overline{\psi}_{A\mu}\gamma_7\gamma^\mu\epsilon^C$; we find the following Ward identity:

$$
-\delta^A_C\,\mathcal{W}\,\overline{\psi}_{A\mu}\gamma^\mu\epsilon^C = \overline{\psi}_{A\mu}\gamma^\mu\left(20\overline{S}^{AB}S_{BC} + 4\overline{N}^{AB}N_{BC} + \frac{1}{4}\overline{M}^{AB}_I M^I_{BC}\right)\epsilon^C\,. \qquad (5.6)
$$

Note that, on the r.h.s. the terms quadratic in the shifts give rise to terms proportional to the current $\overline{\psi}_{A\mu}\gamma^7\gamma^\mu\epsilon^A$, which have no counterpart in the term containing the potential $\mathcal{W}$ and therefore must cancel against each other. This is actually what happens for the first two terms on the r.h.s. of (5.6) taking into account the definition

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of the barred mass matrices given in eq. (5.3). As far as the term $\overline{M}_I^{AB} M_{BC}^I$ is concerned, the same mechanism of cancellation again applies to the terms proportional to $\overline{\psi}_{A\mu}\gamma_7\gamma^\mu\epsilon^A\sigma_C^{rA}$; there is, however, a residual dangerous term of the form

$$\delta_C^A \overline{\psi}_{A\mu}\gamma^\mu\gamma^7 D^I{}_s C_I{}^s\epsilon^C \,. \tag{5.7}$$

One can show that this term vanishes identically owing to the non-trivial relation

$$D_t^I C_I^t = 0 \,. \tag{5.8}$$

Equation (5.8) can be shown to hold using the pseudo-orthogonality relation $L^T\eta L = \eta$ among the coset representatives and the Jacobi identities $C_{I[JK}C_{L]MN} = 0$, $\epsilon_{r[st}\epsilon_{l]mn} = 0$. This is a non-trivial check of our computation.

Note that, setting

$$\begin{aligned} \mathcal{H} &= \frac{1}{12}(Ae^\sigma + 6me^{-3\sigma}L_{00}) \,, \\ \mathcal{K}_i &= \frac{1}{4}(e^\sigma B_i - 2me^{-3\sigma}L_{0i}) \end{aligned} \tag{5.9}$$

the potential can be written as follows

$$\begin{aligned} \mathcal{W} = &-5\{\mathcal{H}^2 + \mathcal{K}^i\mathcal{K}_i\} + \{\partial_\sigma\mathcal{H}\partial_\sigma\mathcal{H} + \partial_\sigma\mathcal{K}^i\partial_\sigma\mathcal{K}_i\} + \\ &+ 2\{\nabla_{I\alpha}\mathcal{H}\nabla^{I\alpha}\mathcal{H} + \nabla_{I\alpha}\mathcal{K}_i\nabla^{I\alpha}\mathcal{K}^i\} + m^2 e^{-6\sigma}L_{0I}L^{0I} \,, \end{aligned} \tag{5.10}$$

where $\nabla_{I\alpha} \equiv (\nabla_{I0}, \nabla_{Ir})$ denote the derivatives with respect to the "linearized coordinates": that is, using the Maurer-Cartan equations

$$\begin{aligned} \nabla^H L^\Lambda{}_I &= L^\Lambda{}_\alpha P_I^\alpha \,, \\ \nabla^H L^\Lambda{}_\alpha &= L^\Lambda{}_I P_\alpha^I \end{aligned} \tag{5.11}$$

the flat derivative $\nabla_{I\alpha}$ are defined as the coefficient of the coset vielbein $P^{I\alpha}$ in eqs. (5.11). In deriving eqs. (5.4)–(5.10) one has to make use of the following relations which are a straightforward consequence of the definitions (4.8)–(4.11)

$$\begin{aligned} \nabla_{I0}A &= 0 \,, \\ \nabla_{Ir}A &= 3C_{Ir} \,, \\ \nabla_{I0}B_i &= C_{iI} \,, \\ \nabla_{Ir}B_i &= 2\epsilon_{rik}D_{Ik} \,. \end{aligned} \tag{5.12}$$

Expanding the squares in eq. (5.5) the potential can be alternatively written as follows:

$$\begin{aligned} \mathcal{W} = &-e^{2\sigma}\left[\frac{1}{36}A^2 + \frac{1}{4}B^iB_i + \frac{1}{4}(C^I{}_tC_{It} + 4D^I{}_tD_{It})\right] + m^2e^{-6\sigma}\mathcal{N}_{00} - \\ &- me^{-2\sigma}\left[\frac{2}{3}AL_{00} - 2B^iL_{0i}\right] \,, \end{aligned} \tag{5.13}$$

where $\mathcal{N}_{00}$ is the 00 component of the vector kinetic matrix defined as in eq. (3.34).

We now show that a stable supersymmetric extremum of the potential $\mathcal{W}$ is found to be the same as in the case of pure supergravity, that is we get an AdS supersymmetric background only for $g = 3m$.[4] In fact, setting $\partial_\sigma \mathcal{W} = 0$ and keeping only the non-vanishing terms at $\sigma = q^{I\alpha} = 0$, $q^{I\alpha}$ being the flat coordinates (that is the coordinates associated to the flat derivatives, or equivalently the coordinates of the linearized theory) we have

$$\partial_\sigma \mathcal{W} = \left[\frac{1}{18} A^2 e^{2\sigma} - \frac{4}{3} m A L_{00} e^{-2\sigma} + 6m^2 L_{00}^2 e^{-6\sigma}\right]_{\sigma = q^{I\alpha}=0} \tag{5.14}$$

since all the other terms entering the $\partial_\sigma \mathcal{W}$ contain at least one off-diagonal element of the coset representative which vanishes identically when the scalar fields are set equal to zero. Furthermore, from the definition (4.8) and using $L^\Lambda{}_\Sigma(q^I_\alpha = 0) = \delta^\Lambda_\Sigma$, we find:

$$A(q^I_\alpha = 0) = 6g\,; \qquad L_{00}(q^I_\alpha = 0) = 1 \tag{5.15}$$

so that

$$\partial_\sigma \mathcal{W}|_{\sigma=q=0} = 2g^2 - 8mg + 6m^2 = 0\,. \tag{5.16}$$

As the partial derivatives $(\partial\mathcal{W}/\partial q^{I0})_{\sigma=q=0}$, $(\partial\mathcal{W}/\partial q^{Ir})_{\sigma=q=0}$ are also zero, since they contains at least one off-diagonal coset representative, the condition for the minimum is given by eq. (5.16) which coincides with the equation one obtains for the pure Supergravity case, whose solutions are $g = m$, $g = 3m$.

Using eqs. (4.6), (5.15) and (5.16) one can easily recognize that only the $g = 3m$ solution gives rise to a supersymmetric AdS background.

An important point regarding this lagrangian is that it incorporates the Higgs mechanism for the two-form B as it was first noticed in the pure supergravity case by Romans [4]. Indeed, the field-strength for the SU(2)-singlet vector A^0_μ only appears in the lagrangian in the combination $F^0_{\mu\nu} - mB_{\mu\nu}$. By performing the gauge transformation

$$\begin{aligned} \delta B_{\mu\nu} &= \partial_{[\mu} f_{\nu]}\,, \\ \delta A^0_\mu &= m f_\mu \end{aligned} \tag{5.17}$$

which leaves $\widehat{\mathcal{F}}^0_{\mu\nu}$ invariant, we can choose f in such a way that the field A^0_μ disappears from the theory, leaving $\widehat{\mathcal{F}}^0_{\mu\nu} = -mB_{\mu\nu}$ so that the lagrangian could be rewritten without any reference to the singlet-vector field A^0_μ, but with a massive two-form $B_{\mu\nu}$. Of course this is in complete agreement with what we found in section 2 at the level of the F.D.A. on which the theory is based.

Incidentally, we note that besides the Ward identity (5.6) there are further supersymmetry Ward identities which relate the gradient of the fermionic shifts with themselves and the spin one-half mass-matrices. They are pretty analogous to the

[4]We leave open the possibility of having other extrema for non zero values of the scalar fields.

gradient flow equations studied in ref. [14] for $N = 1$ and $N = 2$ $D = 4$ supergravity. The $D = 6$ gradient flows are the following:

$$
\begin{aligned}
\partial_\sigma S_{AB} &= \mathrm{i}\overline{N}_{AB}\,,\\
\nabla_{I0} S_{AB} &= -\frac{\mathrm{i}}{4} m e^{-3\sigma} L_{0I} \epsilon_{AB} - \frac{\mathrm{i}}{8} C_{Ir} e^{\sigma} \gamma_7 \sigma^r_{AB}\,,\\
\nabla_{Is} S_{AB} &= -\frac{\mathrm{i}}{4} m e^{-3\sigma} L_{0I} \gamma_7 \sigma_{s|AB} + \frac{\mathrm{i}}{8} C_{Is} e^{\sigma} \epsilon_{AB} + \frac{\mathrm{i}}{4} \epsilon_{rst} D^r_I e^{\sigma} \gamma_7 \sigma^t_{AB}\,,\\
\partial_\sigma \overline{N}_{AB} &= -\frac{1}{4} X_{AB} - 2\mathrm{i} S_{AB} = -2\overline{N}_{AB} - 3\mathrm{i} S_{AB}\,,\\
\nabla_{I0} N_{AB} &= -\frac{3}{4} m e^{-3\sigma} L_{0I} \epsilon_{AB} + \frac{1}{8} C_{Ir} e^{\sigma} \gamma_7 \sigma^r_{AB}\,,\\
\nabla_{Is} N_{AB} &= -\frac{3}{4} m e^{-3\sigma} L_{0I} \gamma_7 \sigma_{s|AB} + \frac{1}{8} C_{Is} e^{\sigma} \epsilon_{AB} - \frac{1}{4} \epsilon_{rst} D^r_I e^{\sigma} \gamma_7 \sigma^t_{AB}\,,\\
\partial_\sigma \overline{M}^I_{AB} &= -2Y^I_{AB} = 8\mathrm{i}(\sigma^{sC}_B \nabla_{Is} S_{AC} - \gamma_7 \nabla_{I0} S_{AB}) - M_{IAB}\,,
\end{aligned}
\tag{5.18}
$$

The previous formulae can be important in the study of renormalization group flows related to the scalar potential of our theory.

A further issue, which is an important check of all our calculation, is the possibility of computing the masses of the gravitational and vector supermultiplets in the Anti de Sitter background. First we compute the masses of the scalar fields by varying the linearized kinetic terms and the potential of (5.1), after power expansion of $\mathcal{W}$ up to the second order in the scalar fields q^I_α.

We find:

$$
\begin{aligned}
\left(\frac{\partial^2 \mathcal{W}}{\partial \sigma^2}\right)_{\sigma=q=0,\, g=3m} &= 48 m^2\,,\\
\left(\frac{\partial^2 \mathcal{W}}{\partial q^{I0} \partial q^{J0}}\right)_{\sigma=q=0,\, g=3m} &= 8 m^2 \delta^{IJ}\,,\\
\left(\frac{\partial^2 \mathcal{W}}{\partial q^{Ir} \partial q^{Js}}\right)_{\sigma=q=0,\, g=3m} &= 12 m^2 \delta^{IJ} \delta^{rs}\,.
\end{aligned}
\tag{5.19}
$$

The linearized equations of motion become:

$$
\begin{aligned}
\Box\sigma - 24 m^2 \sigma + \cdots &= 0\,,\\
\Box q^{I0} - 16 m^2 q^{I0} + \cdots &= 0\,,\\
\Box q^{Ir} - 24 m^2 q^{Ir} + \cdots &= 0\,.
\end{aligned}
\tag{5.20}
$$

If we use as mass unity the inverse AdS radius, which in our conventions (see eq. (4.19)) is $R^{-2}_{\mathrm{AdS}} = 4m^2$ we get:

$$
\begin{aligned}
m^2_\sigma &= -6\,,\\
m^2_{q^{I0}} &= -4\,,\\
m^2_{q^{Ir}} &= -6\,.
\end{aligned}
\tag{5.21}
$$

In the same way we may compute the masses of all the other fields of the multiplets. One finds the following linearized equations of motion:

$$
\begin{aligned}
\mathrm{i}\gamma^\mu \nabla_\mu \lambda_I^A + 4m^2 \lambda_I^A + \cdots &= 0\,, \\
\mathrm{i}\gamma^\mu \nabla_\mu \chi^A + 4m^2 \chi^A + \cdots &= 0\,, \\
\mathrm{i}\gamma^\mu \nabla_\mu \psi^A + 8m^2 \psi^A + \cdots &= 0\,, \\
\nabla_\mu \widehat{\mathcal{F}}^{\mu\nu} + \cdots &= 0\,, \\
\nabla_\mu \mathcal{H}^{\mu\nu\rho} + \frac{8}{3} m^2 B^{\nu\rho} + \cdots &= 0
\end{aligned}
\tag{5.22}
$$

so that, in units of R_{AdS}^{-2} we get:

$$
\begin{aligned}
m_\psi = 2\,, \qquad & m_\lambda = m_\chi = 1\,, \\
m_{A^I} = m_{A^\alpha} = 0\,, \qquad & m^2_{B_{\mu\nu}} = 2\,.
\end{aligned}
\tag{5.23}
$$

These values should be compared with the results obtained in reference [7] where the supergravity and matter multiplets of the AdS_6 $F(4)$ theory were constructed in terms of the singleton fields of the 5-dimensional conformal field theory, the singletons being given by hypermultiplets transforming in the fundamental of $\mathcal{G} \equiv E_7$. It is amusing to see that the values of the masses of all the fields computed in terms of the conformal dimensions are exactly the same as those given in eqs. (5.21) and (5.23). Indeed, using the relations between E_0 and the masses as given for example in [10]:

$$
\begin{aligned}
m^2 &= E_0\,(E_0 - 5) \qquad \text{for scalars}\,, \\
|m| &= E_0 - \frac{5}{2} \qquad \text{for } \frac{1}{2}\text{-spinors}\,, \\
m^2 &= (E_0 - 1)\,(E_0 - 4) \qquad \text{for vectors}\,, \\
m^2 &= (E_0 - 2)\,(E_0 - 3) \qquad \text{for 2-forms}\,, \\
|m| &= E_0 - \frac{5}{2} \qquad \text{for } \frac{3}{2}\text{-spinors}\,, \\
m^2 &= E_0\,(E_0 - 5) \qquad \text{for graviton}
\end{aligned}
\tag{5.24}
$$

it is immediate to retrieve for the masses of all the fields of the supermultiplets, using table 1, the values appearing in the above linearized equations of motion. This coincidence can be considered as a non-trivial check of the AdS/CFT correspondence in six versus five dimensions.

Let us finally observe that the scalar squared masses in AdS_{d+1} are given by the $\mathrm{SO}(2,d)$ quadratic Casimir [12]

$$
m^2 = E_0(E_0 - d)\,. \tag{5.25}
$$

They are negative in the interval $\frac{d-2}{2} \leq E_0 < d$ (the lower bound corresponding to the unitarity bound i.e. the singleton) and attain the Breitenlohner-Freedman bound [13] when $E_0 = d - E_0$ i.e. at $E_0 = d/2$ for which $m^2 = -d^2/4$. Conformal propagation corresponds to $m^2 = -\frac{d^2-1}{4}$ i.e. $E_0 = \frac{d\pm1}{2}$. This is the case of the dilaton and triplet matter scalars.

Grav. mult.	$g_{\mu\nu}$	ψ_A	A^0_μ	A^r_μ	$B_{\mu\nu}$	χ_A	e^σ
E_0	5	9/2	4	4	4	7/2	3
Vector mult.	A^I_μ	λ^A_I	q^{I0}	q^{Ir}			
E_0	4	7/2	4	3			

Table 1: E_0 values for the gravitational and vector multiplets, respectively.

6. Conclusions

In this paper we have given the so far unknown complete lagrangian (up to four-fermion terms) of the matter coupled $D = 6$ $F(4)$ supergravity.

Besides to fill a gap in the supergravity literature, this is a necessary step in order to perform a complete analysis of the AdS_6/CFT_5 correspondence. Indeed, in ref. [5] such correspondence has been established only as far as the supersymmetric general structure of the vector multiplets and gravitational multiplet is concerned. In particular, it was found that there are two series of unitary irreducible representations in the 5-dimensional superconformal field theory which correspond to a massive tower of short vector multiplets and to a tower of massive graviton multiplets, respectively. The lowest members of the two towers are actually massless, and correspond to the conserved currents of the global flavour $\mathcal{G}$ symmetry of the 5-dimensional conformal field theory and to the stress-tensor multiplet, respectively. However, in that framework it was not possible to determine the $\mathcal{G}$ quantum numbers of the supermultiplets. On the other hand, in ref. [8] it was established that the $F(4)$ supergravity theory can be obtained as a Kaluza-Klein reduction of massive type-IIA supergravity on a background AdS_6 which is fibered with a warped four-sphere. This reduction is related to the horizon geometry of $D4$-branes in a $D8$-brane background, in presence of a $D0$-brane. In the same reference the warped metric for the reduction was obtained. In particular, in a recent paper [15], solutions for Romans' 6-dimensional gauged supergravity, related to D-branes interpretations, were found and then lifted up to ten dimensions.

The knowledge of the $F(4)$ lagrangian in $D = 6$ together with that of the warped metric for the compactifiction, allows one to perform in principle an analysis of the complete Kaluza-Klein spectrum, which would exhibit, besides the supermultiplet structure, also the $\mathcal{G}$ flavour quantum numbers. This program is left to a future investigation.

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A. The superspace curvature and the solution of Bianchi identities.

As it has been stressed in the text, the supersymmetry transformation laws for the physical fields have been obtained by solving the Bianchi identities in superspace. In our case we follow the particular approach developed in [9], which is substantially equivalent to the other existing superspace approaches, the only difference being the more precise group-theoretical assessment of the starting points.

The first step is to find the solution of Bianchi identities (3.17) in absence of gauging.

The solution can be obtained as follows: first of all one requires that the expansion of the curvatures along an intrinsic p-form basis in superspace generated by V^a and ψ is given in terms only of the physical fields. This insures that no new degree of freedom is introduced in the theory. This property has been also referred to as "rheonomy" in the literature.

Secondly one writes down such expansion in a form which is compatible with all the symmetries of the theory, that is: covariance under SU(2) R-symmetry, Lorentz transformations and reparametrizations of the scalar manifold. Besides one has to take into account the invariance under the following rigid rescalings of the fields (and their corresponding curvatures):

$$\begin{aligned}
(\omega^{ab}, \phi^i, \sigma) &\longrightarrow (\omega^{ab}, \phi^i, \sigma)\,, \\
(V^a, A^\Lambda) &\longrightarrow \ell(V^a, A^\Lambda)\,, \\
B_{\mu\nu} &\longrightarrow \ell^2 B_{\mu\nu}\,, \\
\psi_A &\longrightarrow \ell^{1/2}\psi_A\,, \\
(\lambda^{IA}, \chi_A) &\longrightarrow \ell^{-1/2}(\lambda^{IA}, \chi_A)\,,
\end{aligned} \tag{A.1}$$

Indeed these rescalings and the corresponding ones for the curvatures leave invariant the definitions of the curvatures (in particular the F.D.A.) and the Bianchi identities.

Furthermore, the parametrizations of the curvatures must scale under the $O(1,1)$ duality group discussed in section 3 in a uniform way.

Taking into account all these constraints, one finds that the parametrizations of the curvatures of all the fields which solve the Bianchi identities of the ungauged curvatures (3.15) have the following form (for notations and definitions see the text):

$$\begin{aligned}
T^a &= 0\,, \\
H &= H_{abc}V^a \wedge V^b \wedge V^c + 4\mathrm{i}e^{-2\sigma}\overline{\psi}_A\gamma_7\gamma_{ab}\chi^A \wedge V^a \wedge V^b\,, \\
\widehat{F}^\Lambda &= \widehat{F}^\Lambda_{ab}V^a \wedge V^b + 2e^\sigma L^\Lambda_0\overline{\psi}_A\gamma_7\gamma_a\chi^A \wedge V^a + 2e^\sigma L^\Lambda_{(AB)}\overline{\psi}^A\gamma_a\chi^B \wedge V^a - \\
&\quad - e^\sigma L^\Lambda_I\overline{\psi}_A\gamma_a\lambda^{IA} \wedge V^a\,,
\end{aligned}$$

$$
\begin{aligned}
\rho_A &= \rho_{A|ab} V^a \wedge V^b + \frac{1}{16} e^{-\sigma} \left(T_{ab}\epsilon_{AB}\gamma_7 - T_{(AB)|ab}\right) \left(\gamma^{abc} - 6\delta^{ca}\gamma^b\right) \psi^B \wedge V_c + \\
&\quad + \frac{\mathrm{i}}{32} e^{2\sigma} H_{abc}\gamma_7 \left(\gamma^{dabc} - 3\delta^{da}\gamma^{bc}\right) \psi_A \wedge V_d + \rho_{A(2\psi)} + \rho_A^{(3f)}\,, \\
\nabla\chi_A &= \nabla_a\chi_A V^a + \frac{\mathrm{i}}{2}\gamma^a\partial_a\sigma\psi_A + \frac{\mathrm{i}}{16} e^{-\sigma} \left(T_{ab}\epsilon_{AB}\gamma_7 + T_{(AB)|ab}\right)\gamma^{ab}\psi^B + \\
&\quad + \frac{1}{32} e^{2\sigma} H_{abc}\gamma_7\gamma^{abc}\psi_A + \nabla\chi_A^{(3f)}\,, \\
\nabla\lambda^{IA} &= \nabla_a\lambda^{IA} V^a + \mathrm{i}P_i^{I0}\partial_a\phi^i\gamma_7\gamma^a\psi^A - \mathrm{i}P_i^{I(AB)}\partial_a\phi^i\gamma^a\psi_B + \frac{\mathrm{i}}{2} e^{-\sigma} T^I_{ab}\gamma^{ab}\psi^A + \\
&\quad + \nabla\lambda^{IA(3f)}\,, \\
d\sigma &= \partial_a\sigma V^a + \overline{\chi}_A\psi^A\,, \\
P^{I0} &= P_i^{I0}\partial_a\phi^i V^a + \frac{1}{2}\overline{\lambda}^I_A\gamma_7\psi^A\,, \\
P_{I(AB)} &= P_{I(AB)i}\partial_a\phi^i V^a - \overline{\lambda}_{IA}\psi_B\,,
\end{aligned}
\tag{A.2}
$$

where:

$$
\begin{aligned}
\rho_{A(2\psi)} &= \frac{1}{4}\gamma_7\psi_A\overline{\chi}^C\gamma^7\psi_C - \frac{1}{2}\gamma_a\psi_A\overline{\chi}^C\gamma^a\psi_C + \frac{1}{2}\gamma_7\gamma_a\psi_A\overline{\chi}^C\gamma^7\gamma^a\psi_C - \\
&\quad - \frac{1}{8}\gamma_{ab}\psi_A\overline{\chi}^C\gamma^{ab}\psi_C - \frac{1}{8}\gamma_7\gamma_{ab}\psi_A\overline{\chi}^C\gamma^7\gamma^{ab}\psi_C + \frac{1}{4}\psi_A\overline{\chi}^C\psi_C
\end{aligned}
\tag{A.3}
$$

and $\rho_A^{(3f)}$, $\nabla\chi_A^{(3f)}$, $\nabla\lambda^{IA(3f)}$ denote terms constructed out in terms of one fermionic vielbein ψ and two spin 1/2 — fermions, namely of the form: $\psi\chi\chi$, $\psi\chi\lambda$, $\psi\lambda\lambda$ which have not been computed.[5]

Let us make a few comments about this solution. An important point is the presence of the term $\rho_{A(2\psi)}$ in the gravitino curvature ρ_A. Indeed, this term is essential in order to solve in a consistent way the previous Bianchi identities, already in the pure supergravity sector, that is setting the vector multiplets to zero. The pure supergravity Bianchi identities take, in this case, the following form:

$$R^{ab}V_b - i\overline{\psi}_A\gamma^a\rho_B\epsilon^{AB} = 0\,, \tag{A.5}$$
$$\mathcal{D}R^{ab} = 0\,, \tag{A.6}$$
$$dH + 4e^{-2\sigma}d\sigma\,\overline{\psi}_A\gamma_7\gamma_a\psi_B\epsilon^{AB}V^a + 4e^{-2\sigma}\overline{\psi}_A\gamma_7\gamma_a\rho_B\epsilon^{AB}V^a = 0\,, \tag{A.7}$$
$$DF + id\sigma e^{\sigma}\,\overline{\psi}_A\gamma_7\psi_B\,\epsilon^{AB} - 2ie^{\sigma}\,\overline{\psi}_A\gamma_7\rho_B\,\epsilon^{AB} = 0\,, \tag{A.8}$$
$$DF^r + id\sigma e^{\sigma}\,\overline{\psi}_A\psi_B\,\sigma^{rAB} - 2ie^{\sigma}\,\overline{\psi}_A\rho_B\,\sigma^{rAB} = 0\,, \tag{A.9}$$

[5]Note that the flat superspace derivatives along a set of vielbein V^a's are what in component formalism are called the supercovariant field-strengths. Indeed, if we project the curvatures (A.2) along the space-time differentials dx^μ's we find, e.g. from the second equation:

$$H_{\mu\nu\rho} = H_{abc}V^a_\mu V^b_\nu V^c_\rho + 4\mathrm{i}e^{-2\sigma}\overline{\psi}_{A[\mu}\gamma_7\gamma_{\nu\rho]}\chi^A \tag{A.4}$$

and $\widetilde{H}_{\mu\nu\rho} \equiv H_{abc}V^a_\mu V^b_\nu V^c_\rho$ is the supercovariant field-strength. The same observation applies to all the flat derivatives appearing in eqs. (A.2).

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$$D\rho_A + \frac{1}{4}R^{ab}\gamma_{ab}\psi_A - \frac{i}{2}\, g\, \sigma_{rAB}F^r\psi^B = 0\,, \tag{A.10}$$

$$DR_A + \frac{1}{4}R^{ab}\gamma_{ab}\chi_A - \frac{i}{2}\, g\, \sigma_{rAB}F^r\chi^B = 0\,, \tag{A.11}$$

$$d^2\sigma = 0\,. \tag{A.12}$$

The necessity of the $\rho_{A(2\psi)}$ term in the gravitino curvature can indeed be ascertained when one tries to solve the previous Bianchi identities at the highest level, that is in the sector containing $\psi \wedge \psi \wedge \psi$ (3 fermionic vielbeins). Indeed, it is not difficult to verify that the sector $(V\chi\psi\psi\psi)$ of eq. (A.7) and the sectors $(\chi\psi\psi\psi)$ of eqs. (A.5), (A.8) and (A.9) do not close unless we add a suitable $\rho_{A(2\psi)}$ as given in eq. (A.3) in the gravitino curvature. To arrive at this result requires a lengthy and cumbersome computation since one has to use several times the Fierz identities between three-ψ one-forms, discussed in appendix C.

We stress that the verification of the closure of the Bianchi identities at the three-ψ level is quite essential; this is to be contrasted with the fact that, in the analysis of the Bianchi identities, we neglected the three fermions terms $\rho^{(3f)}$ in the gravitino curvature containing only one ψ, that is terms of the form $\psi\chi\chi$, $\psi\lambda\lambda$, $\psi\lambda\chi$. Indeed it is well known that once the Bianchi identities have been satisfied in the highest sectors with three or two ψ, then they automatically close (on shell) in the sectors containing only one ψ. In this sense, since the three-ψ's terms in superspace are terms of the form $\psi\psi\epsilon$ on space-time, and the latter were neglected in ref. [4], we may say that our analysis proves the consistency of Romans' construction of pure $F(4)$ supergravity also at the three-fermion level.

For the benefit of the reader not familiar with the Superspace Bianchi identities, we recall that the determination of the superspace curvatures enables us to write down the space-time supersymmetry transformation laws. Indeed, from the superspace point of view, a supersymmetry transformation is a Lie derivative along the tangent vector:

$$\epsilon = \bar{\epsilon}^A\, \vec{D}_A\,, \tag{A.13}$$

where the basis tangent vector $\vec{D}_A$ is dual to the gravitino 1-form:

$$\vec{D}_A\left(\psi^B\right) = \delta^B_A \mathbf{1}\,, \tag{A.14}$$

where $\mathbf{1}$ is the unit in spinor space.

Denoting by μ^I and R^I the set of 1-forms and 2-form potential $(V^a,\, \psi_A,\, A^\Lambda, B)$ and of 2-form and 3-form curvatures $(T^a,\, \rho_A,\, F^\Lambda,\, H)$, respectively, one has:

$$\ell_\epsilon \mu^I = \left(i_\epsilon d + d i_\epsilon\right)\mu^I \equiv \left(D\epsilon\right)^I + i_\epsilon R^I\,, \tag{A.15}$$

where D is the derivative covariant with respect to the $N = 2$ Poincaré superalgebra and i_ϵ is the contraction operator along the tangent vector ϵ.

In our case:

$$
\begin{aligned}
(D\epsilon)^a &= -\mathrm{i}\left(\overline{\psi}_A \gamma^a \epsilon^A\right) \,, \\
(D\epsilon_A)^\alpha &= \nabla \epsilon_A^\alpha \,, \\
(D\epsilon)^\Lambda &= 0
\end{aligned}
\tag{A.16}
$$

(here α is a spinor index)

For the 0-forms which we denote shortly as $\nu^I \equiv (q^u,\, \sigma,\, \lambda^{IA}, \chi_A)$ we have the simpler result:

$$
\ell_\epsilon = i_\epsilon d\nu^I = i_\epsilon \left(\nabla \nu^I - \text{connection terms}\right) , \tag{A.17}
$$

Using the parametrizations given for R^I and $\nabla \nu^I$ and identifying δ_ϵ with the restriction of ℓ_ϵ to space-time it is immediate to find the $N = 2$ susy laws for all the fields. The explicit formulae for the ungauged case are given by the eqs. (3.18)–(3.26) of the text.

Let us now consider the gauged Bianchi identities.

In the gauged theory, the curvatures are defined formally as in eqs. (3.15) provided we use the gauged connections and vielbeins of the scalar manifold: $\widehat{\Omega} = L^{-1}\nabla L$, defined in eqs. (4.1)–(4.4), instead of the ungauged ones: $\Omega = L^{-1}dL$.

Correspondingly, the new gauged Bianchi identities become:

$$
\begin{aligned}
R^{ab}V_b - i\overline{\psi}_A \gamma^a \rho_B \epsilon^{AB} &= 0 \,, \\
\mathcal{D}R^{ab} &= 0 \,, \\
dH + 4e^{-2\sigma} d\sigma\, \overline{\psi}_A \gamma_7 \gamma_a \psi_B \epsilon^{AB} V^a + 4e^{-2\sigma} \overline{\psi}_A \gamma_7 \gamma_a \rho_B \epsilon^{AB} V^a &= 0 \,, \\
DF^\Lambda + id\sigma e^\sigma \overline{\psi}^A \gamma_7 \psi^B L^\Lambda_{[AB]} + id\sigma e^\sigma \overline{\psi}^A \psi^B L^\Lambda_{(AB)} - & \\
- 2ie^\sigma \overline{\psi}^A \gamma_7 \rho^B L^\Lambda_{[AB]} - 2ie^\sigma \overline{\psi}^A \rho^B L^\Lambda_{(AB)} + & \\
+ ie^\sigma L^\Lambda{}_I \overline{\psi}^A \psi^B \widehat{P}^I_{(AB)} + ie^\sigma L^\Lambda{}_I \overline{\psi}^A \gamma_7 \psi^B \widehat{P}^I_{[AB]} &= 0 \,, \\
D\rho_A + \frac{1}{4} R^{ab} \gamma_{ab} \psi_A - \frac{i}{2} \sigma^r_{AB} \left(\frac{1}{2} \epsilon^{rst} \widehat{\mathcal{R}}^{st} + i\gamma_7 \widehat{\mathcal{R}}_{r0}\right) \psi^B &= 0 \,, \\
D^2 \chi_A + \frac{1}{4} R^{ab} \gamma_{ab} \chi_A - \frac{i}{2} \sigma^r_{AB} \left(\frac{1}{2} \epsilon^{rst} \widehat{\mathcal{R}}^{st} + i\gamma_7 \widehat{\mathcal{R}}_{r0}\right) \chi^B &= 0 \,, \\
d^2\sigma &= 0 \,, \\
D^2 \lambda^I_A + \frac{1}{4} R^{ab} \gamma_{ab} \lambda^I_A - \frac{i}{2} \sigma^r_{AB} \left(\frac{1}{2} \epsilon^{rst} \widehat{\mathcal{R}}^{st} + i\gamma_7 \widehat{\mathcal{R}}_{r0}\right) \lambda^{IB} - \widehat{\mathcal{R}}^I_J \lambda^J_A &= 0 \,, \\
D\widehat{P}^I_r &= \left(L^{-1} \widetilde{\mathcal{F}} L\right)^I_r \,, \\
D\widehat{P}^I_0 &= \left(L^{-1} \widetilde{\mathcal{F}} L\right)^I_0 \,,
\end{aligned}
\tag{A.18}
$$

where all the "hatted" quantities have been defined in the text and the covariant derivatives are now covariant also with respect to the gauge group $\mathrm{SU}(2) \otimes \mathcal{G}$. As it happens in all gauged supergravities, the new solution of the Bianchi Identities differs from the old one in the following aspects:

1. The vector field-strengths are now non abelian.

2. The derivatives have to be made covariant also with respect to the gauge group, so that the non-abelian field-strengths appearing on the l.h.s. of the old parametrizations (A.2) now contain the "hatted" connections and vielbeins of the scalar manifold.

3. The parametrizations of the fermionic curvatures ρ_A, $\nabla\chi_A$, $\nabla\lambda^{IA}$ contain extra terms $S_{AB}\gamma_a\psi V^a$, $N_{AB}\psi^B$, $M^I_{AB}\psi^B$ which are proportional to the gauge coupling constants g, g'.

As it has been explained in the text, there is however a more general solution of the B.I. involving a second "gauging" in terms of the mass parameter m so that the previous fermionic shifts can also acquire extra terms proportional to m. The computation of the complete shifts is actually quite cumbersome since we have to use several times the relevant Fierz-identities quoted in appendix C and further decompose the relevant structures in SU(2) irreducible fragments. The final result for the shifts is given in the text (see eqs. (4.16)). The resulting supersymmetry transformation laws on space-time can be finally obtained as explained in the ungauged case and are given by eq. (4.17) of the text.

B. The lagrangian from the geometric approach

In appendix A we have outlined how to recover the supersymmetry transformation laws for the physical fields of the matter coupled $F(4)$ theory from the solution of Bianchi identities in superspace. Since the closure of Bianchi identities is true only on the mass-shell, the equations of motion for all the fields are also implicitely given, and from them one could in principle reconstruct the lagrangian. However, this procedure would be quite cumbersome. We therefore prefered to work out the space-time lagrangian from a geometric lagrangian in superspace, whose construction in the geometric (rheonomic) approach is straightforward. This appendix gives a short account of its derivation.

In the geometric approach the superspace action for the theory is a six-form in superspace[6] integrated on a 6-dimensional (bosonic) hypersurface $\mathcal{M}^6$ locally embedded in $\mathcal{M}^{6|16}$:

$$\mathcal{A} = \int_{\mathcal{M}^6 \subset \mathcal{M}^{6|16}} \mathcal{L} \,. \tag{B.1}$$

[6]The superspace we are considering, $\mathcal{M}^{6|16}$, contains 6 space-time directions and 16 ($N = 2$) fermionic directions.

It contains the fields of the theory through external forms on $\mathcal{M}^{6|16}$, using only diffeomorphism-invariant operations of external algebra, namely the exterior derivative d and the wedge product $\wedge$ (we therefore never introduce the Hodge duality operator, which depends on the choice of the hypersurface of integration). We then make use of a generalized variational principle ($\delta\mathcal{A} = 0$), which provides superspace equations of motion that are 4-form, 5-form or 6-form equations independent from the particular hypersurface $\mathcal{M}^6 \subset \mathcal{M}^{6|16}$ on which we integrate. These superspace equations of motion can be analyzed along the p-form basis. The components of the equations along bosonic vielbeins give the differential equations for the fields which, identifying $\mathcal{M}^6$ with space-time, are the ordinary equations of motion of the theory. The components of the same equations along p-forms containing at least one gravitino ("outer components") instead, according to the priciple of rheonomy, must be all expressed in terms of the supercovariant internal components (components along the bosonic vielbeins basis). Actually if we have already solved the Bianchi identities this requirement is equivalent to identify the outer components of the curvatures obtained from the variational principle with those obtained from the Bianchi identities.

There are simple rules which can be used in order to write down the most general lagrangian compatible with this requirement.

Actually one writes down the most general 6-form as a sum of terms with indeterminate coefficients in such a way that $\mathcal{L}$ be a scalar with respect to all the symmetry transformations of the theory. In order to avoid the use of the Hodge operator (which would destroy the independence of the variational equations from the particular hypersurface of integration) the kinetic terms of the lagrangian have been written in first-order formalism. Specifically one introduces auxiliary 0-forms namely $\mathbf{H}_{abc}$, $\mathbf{F}^{\Lambda}_{ab}$, $\mathbf{P}^{IAB}_{a}$, $\mathbf{\Sigma_a}$, whose variational equations identify them with H_{abc}, $\widehat{F}^{\Lambda}_{ab}$, $P^{IAB}_{i}\partial_a\phi^i \equiv P^{IAB}_{a}$, $\partial_a\sigma$ appearing in eqs. (A.2) of appendix A. Also the spin connection ω^{ab} has to be treated as an independent field: indeed the terms in the lagrangian containing explicitely the torsion T^a have been chosen in such a way that the equation of motion of ω^{ab} gives $T^a = 0$.

Varying the action and comparing the outer equations of motion with the actual solution of the Bianchi identities one then fixes all the undetermined coefficients except the coefficients of terms that are proportional to $V^a \wedge V^b \wedge \cdots V^f \epsilon_{abcdef}$. Indeed, after variation, these last terms do not contain any fermionic vielbein ψ and appear therefore in the space-time equations of motion.These undetermined coefficients, however, can be retrieved by the request that the superspace lagrangian be invariant under supersymmetry transformation, that is by implementing the condition:

$$\delta_\epsilon \mathcal{L} = i_\epsilon d\mathcal{L} = 0\,, \tag{B.2}$$

where i_ϵ denotes contraction of the generator of supersymmetry $\epsilon = \epsilon^A Q_A$ on the form.

Let us perform the steps previously indicated. The most general 6-form lagrangian, up to four-fermions terms, has the following form:

$$
\begin{aligned}
\mathcal{L}^{(g,g',m)}_{(D=6,N=2)} = {} & \mathcal{R}^{ab}\wedge V^c\cdots\wedge V^f\epsilon_{a\cdots f}+\left(p_1\overline{\lambda}^I_A\gamma^a\nabla\lambda^A_I+p_2\overline{\chi}_A\gamma_a\nabla\chi^A\right)\wedge\\
& \wedge V^b\cdots\wedge V^f\epsilon_{a\cdots f}+a_1\overline{\psi}_A\gamma_7\gamma^{abc}\rho^A\wedge V^a\cdots\wedge V^c+\\
& +\left[b_3\boldsymbol{\Sigma}_{\mathbf{a}}\left(d\sigma-\overline{\chi}_A\psi^A\right)+b_1\mathbf{P}^{I0}_a\left(\widehat{P}_{I0}-\frac{1}{2}\overline{\lambda}_{IA}\gamma_7\psi^A\right)+\right.\\
& \left.\quad+b_2\mathbf{P}^{I(AB)}_a\left(\widehat{P}_{I(AB)}+\overline{\lambda}_{IA}\psi_B\right)\right]\wedge V_b\cdots\wedge V_f\epsilon^{a\cdots f}-\\
& -\frac{1}{12}\left[b_1\mathbf{P}^I_m\mathbf{P}^m_I+b_2\mathbf{P}^{I(AB)}_m\mathbf{P}^m_{I(AB)}+b_3\boldsymbol{\Sigma}_{\mathbf{m}}\boldsymbol{\Sigma}^{\mathbf{m}}\right]V_a\cdots\wedge V_f\epsilon^{a\cdots f}+\\
& +d_1e^{-2\sigma}\mathcal{N}_{\Lambda\Sigma}\left[\mathbf{F}^\Lambda_{ab}\left(\widehat{F}^\Sigma-2e^\sigma L^\Sigma_0\overline{\psi}_A\gamma_7\gamma_\ell\chi^A\wedge V^\ell-\right.\right.\\
& \qquad\left.-2e^\sigma L^\Sigma_{(AB)}\overline{\psi}^A\gamma_\ell\chi^B\wedge V^\ell+e^\sigma L^\Sigma_I\overline{\psi}_A\gamma_\ell\lambda^{IA}\wedge V^\ell\right)\wedge\\
& \qquad\left.\wedge V_c\cdots\wedge V_f\epsilon^{a\cdots f}-\frac{1}{30}\mathbf{F}^\Lambda_{\ell m}\mathbf{F}^{\Sigma\ell m}V^a\cdots\wedge V^f\epsilon_{a\cdots f}\right]+\\
& +f_1e^{4\sigma}\mathbf{H}_{abc}\left[H-4\mathrm{i}e^{-2\sigma}\overline{\psi}_A\gamma_7\gamma_{\ell m}\chi^A\wedge V^\ell\wedge V^m\right]\wedge V_d\cdots\wedge V_f\epsilon^{a\cdots f}-\\
& -\frac{1}{40}f_1e^{4\sigma}\mathbf{H}_{\ell mn}\mathbf{H}^{\ell mn}V_a\cdots\wedge V_f\epsilon^{a\cdots f}+\\
& +\left[g_1d\sigma\overline{\psi}_A\gamma_{ab}\chi^A+g_2\widehat{P}^I\overline{\psi}_A\gamma_7\gamma_{ab}\lambda^A_I+g_3\widehat{P}^I_{(AB)}\overline{\psi}^A\gamma_7\gamma_{ab}\lambda^B_I\right]\wedge\\
& \wedge V_c\cdots\wedge V_f\epsilon^{a\cdots f}+\\
& +e^{-\sigma}\mathcal{N}_{\Lambda\Sigma}\widehat{F}^\Lambda\left[\left(h_1L^\Sigma_0\overline{\psi}_A\gamma_{ab}\psi^A+h_2L^\Sigma_{(AB)}\overline{\psi}^A\gamma_7\gamma_{ab}\psi^B\right)\wedge V^a\wedge V^b+\right.\\
& \qquad+\left(h_3L^\Sigma_0\overline{\psi}_A\gamma_7\gamma^{abc}\chi^A+h'_3L^\Sigma_{(AB)}\overline{\psi}^A\gamma^{abc}\chi^B+\right.\\
& \qquad\quad\left.+h_4L^\Sigma_I\overline{\psi}_A\gamma^{abc}\lambda^{IA}\right)\wedge V^d\cdots\wedge V^f\epsilon_{a\cdots f}+\\
& \qquad+\left(h_5L^\Sigma_0\overline{\lambda}_{IA}\gamma_7\gamma^{ab}\lambda^{IA}+h'_5L^\Sigma_{(AB)}\overline{\lambda}^A_I\gamma^{ab}\lambda^{IB}+\right.\\
& \qquad\quad\left.+h_6L^\Sigma_0\overline{\chi}_A\gamma_7\gamma^{ab}\chi^A+h'_6L^\Sigma_{(AB)}\overline{\chi}^A\gamma^{ab}\chi^B\right)\wedge\\
& \qquad\left.\wedge V^c\cdots\wedge V^f\epsilon_{a\cdots f}\right]+\\
& +e^{2\sigma}H\wedge\left[p'_1\overline{\psi}_A\gamma_a\psi^A\wedge V^a+p'_2\overline{\psi}_A\gamma_{ab}\chi^A\wedge V^a\wedge V^b+\right.\\
& \qquad\left.+p_5\overline{\chi}_A\gamma_{abc}\chi^AV^a\cdots\wedge V^c\right]+\\
& +T_a\wedge\left[\left(\ell_1\overline{\lambda}^A_I\gamma_7\gamma_{bcd}\lambda^{IA}+\ell_2\overline{\chi}_A\gamma_7\gamma_{bcd}\chi^A\right)V^a\cdots\wedge V^d+\right.\\
& \qquad\left.+\ell_3\overline{\psi}_A\gamma_7\gamma_b\psi^A\wedge V^a\wedge V^b\right]+\\
& +kB\wedge\left[\eta_{\Lambda\Sigma}\left(\widehat{F}^\Lambda+\mathrm{i}e^\sigma L^\Lambda_0\overline{\psi}_A\gamma_7\psi^A+\mathrm{i}e^\sigma L^\Lambda_{(AB)}\overline{\psi}^A\psi^B\right)\wedge\right.
\end{aligned}
$$

$$
\begin{aligned}
&\wedge\left(\widehat{F}^{\Sigma}+\mathrm{i}e^{\sigma}L_0^{\Sigma}\overline{\psi}_C\gamma_7\psi^C+\mathrm{i}e^{\sigma}L^{\Sigma}_{(CD)}\overline{\psi}^C\psi^D\right)+\\
&+\ mB\delta^0_{\Lambda}\wedge\left(\widehat{F}^{\Lambda}+\mathrm{i}e^{\sigma}L_0^{\Lambda}\overline{\psi}_A\gamma_7\psi^A+\mathrm{i}e^{\sigma}L^{\Lambda}_{(AB)}\overline{\psi}^A\psi^B\right)+\\
&+\frac{1}{3}m^2B\wedge B\Big]+\\
+&\left[\delta_1\overline{\psi}_A\gamma^{ab}\overline{S}^{AB}\psi_B+\left(\delta_2\overline{\psi}_A\gamma^a\overline{N}^{AB}\chi_B+\delta_3\overline{\psi}_A\gamma^a\overline{M}_I^{AB}\lambda_B^I\right)\wedge V^b\right]\times\\
\times&\, V^c\cdots\wedge V^f\epsilon_{a\cdots f}+\delta_4\left(\overline{\chi}^AX_{AB}\chi^B+\overline{\chi}^AY^I_{AB}\lambda^B_I+\overline{\lambda}^I_AZ^{AB}_{IJ}\lambda^J_B\right)\times\\
\times&\, V^a\cdots\wedge V^f\epsilon_{a\cdots f}-\mathcal{W}(\sigma,\,\phi^i;g,g',m)V^a\cdots\wedge V^f\epsilon_{a\cdots f}+\\
+&\,\mathcal{L}_{4\,\mathrm{fermions}}\,.
\end{aligned}
\tag{B.3}
$$

The mass matrices for the spin 1/2 fermions, X_{AB}, Y^I_{AB}, Z^{AB}_{IJ}, were computed through the request of supersymmetry invariance of the superspace lagrangian and are given by (5.4).

The coefficients appearing in (B.3) have been found, from the superspace variational equations, to be:

$$
\begin{array}{llllll}
a_1=-8\,; & b_1=-\frac{2}{5}\,; & b_2=\frac{1}{5}\,; & b_3=\frac{8}{5}\,; & d_1=-\frac{1}{2}\,; & f_1=\frac{1}{4}\,;\\
p_1=-\frac{\mathrm{i}}{10}\,; & p_2=-\frac{8}{5}\mathrm{i}\,; & k=\frac{3}{2}\,; & h_1=6\,; & h_2=-6\,; & h_3=-\frac{2}{3}\,;\\
h'_3=-\frac{2}{3}\,; & h_4=-\frac{1}{3}\,; & h_5=\frac{\mathrm{i}}{16}\,; & h'_5=\frac{\mathrm{i}}{16}\,; & h_6=\frac{\mathrm{i}}{2}\,; & h'_6=-\frac{\mathrm{i}}{2}\,;\\
p'_1=3\mathrm{i}\,; & p'_2=-6\,; & p_5=-2\mathrm{i}\,; & g_1=-4\,; & g_2=-\frac{1}{2}\,; & g_3=-\frac{1}{2}\,;\\
\ell_1=3\,; & \ell_2=-48\,; & \ell_3=-12\,; & \delta_1=4\mathrm{i}\,; & \delta_2=\frac{16}{5}\mathrm{i}\,; & \delta_3=-\frac{\mathrm{i}}{5}\,;\\
\delta_4=\frac{2}{15}\,. & & & & &
\end{array}
\tag{B.4}
$$

In order to obtain the space-time lagrangian the last step to perform is the restriction of the 6-form lagrangian from superspace to space-time. This corresponds to restrict all the terms to the particular hypersurface $\mathcal{M}^6$ with $\theta=0$, $d\theta=0$. In practice one first goes to the second order formalism by identifying the auxiliary 0-form fields as explained before. Then one expands all the forms along the dx^μ differentials and restricts the superfields to their lowest ($\theta=0$) component. Finally the coefficients of:

$$
dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_6}=\frac{\epsilon^{\mu_1\cdots\mu_6}}{\sqrt{g}}\left(\sqrt{g}d^6x\right)
\tag{B.5}
$$

give the lagrangian density written in section 3. The overall normalisation of the space-time action has been chosen such as to be the standard one for the Einstein term.

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C. $D = 6,\ N = 2$ Fierz identities for pseudo-Majorana spinors

All the Fierz identities can be derived from the following fundamental bilinear identity:

$$\psi_B\overline{\psi}_A = \frac{1}{8}\overline{\psi}_A\psi_B 1 - \frac{1}{8}\overline{\psi}_A\gamma^7\psi_B\gamma_7 + \frac{1}{8}\overline{\psi}_A\gamma^a\psi_B\gamma_a + \frac{1}{8}\overline{\psi}_A\gamma^7\gamma^a\psi_B\gamma_7\gamma_a + \\ - \frac{1}{16}\overline{\psi}_A\gamma^{ab}\psi_B\gamma_{ab} + \frac{1}{16}\overline{\psi}_A\gamma^7\gamma^{ab}\psi_B\gamma_7\gamma_{ab} - \frac{1}{48}\overline{\psi}_A\gamma^{abc}\psi_B\gamma_{abc}\,. \qquad \text{(C.1)}$$

Using (C.1) and the symmetry properties of the gamma-matrices it is immediate to check the following quadri-linear identity:

$$\overline{\psi}_A\gamma_a\psi_B\overline{\psi}_C\gamma^7\gamma^a\psi_D\epsilon^{AB}\epsilon^{CD} = 0\,. \qquad \text{(C.2)}$$

In order to obtain three-linear identities, we write down all the possible three-linear terms with no explicit Lorentz index and with a given SU(2) tensor structure. We have:

$$\begin{aligned}
A^1_A &= \gamma_7\psi_A\overline{\psi}_B\gamma^7\psi_C\epsilon^{BC}\,,\\
A^2_A &= \gamma_a\psi_A\overline{\psi}_B\gamma^a\psi_C\epsilon^{BC}\,,\\
A^3_A &= \gamma_7\gamma_a\psi_A\overline{\psi}_B\gamma^7\gamma^a\psi_C\epsilon^{BC}\,,\\
A^4_A &= \gamma_{ab}\psi_A\overline{\psi}_B\gamma^{ab}\psi_C\epsilon^{BC}\,,\\
S^1_A &= \psi^B\overline{\psi}_A\psi_B\,,\\
S^2_A &= \gamma_{abc}\psi^B\overline{\psi}_A\gamma^{abc}\psi_B\,,\\
S^3_A &= \gamma_7\gamma_{ab}\psi^B\overline{\psi}_A\gamma^7\gamma^{ab}\psi_B\,,\\
S^1_{(ABC)} &= \psi_{(A}\overline{\psi}_B\psi_{C)}\,,\\
S^2_{(ABC)} &= \gamma_{abc}\psi_{(A}\overline{\psi}_B\gamma^{abc}\psi_{C)}\,,\\
S^3_{(ABC)} &= \gamma_7\gamma_{ab}\psi_{(A}\overline{\psi}_B\gamma^7\gamma^{ab}\psi_{C)}\,.
\end{aligned}$$

A group-theoretical analysis. together with numerical Mathematica computations, gives the following Fierz identities:

$$A^3_A = A^2_A\,, \qquad \text{(C.3)}$$

$$A^4_A = 6A^1_A - 4A^2_A\,, \qquad \text{(C.4)}$$

$$S^1_A = \frac{1}{2}A^2_A - \frac{1}{2}A^1_A\,, \qquad \text{(C.5)}$$

$$S^3_A = -3A^1_A - 3A^2_A\,, \qquad \text{(C.6)}$$

$$S^3_{(ABC)} = 6S^1_{(ABC)}\,, \qquad \text{(C.7)}$$

$$S^2_{(ABC)} = -24S^1_{(ABC)}\,. \qquad \text{(C.8)}$$

In particular, for the purpose of solving the vectors Bianchi identities at the three ψ-level, one can derive from eqs. (C.3)–(C.6) the following useful identity:

$$4\overline{\chi}_{(A}\gamma_a\psi_{B)}\overline{\psi}_C\gamma^A\psi^C - 6\overline{\chi}_C\psi^C\overline{\psi}_{(A}\psi_{B)} + \overline{\chi}_C\gamma_7\gamma_{ab}\psi^C\overline{\psi}_{(A}\gamma^7\gamma^{ab}\psi_{B)} = 0\,. \qquad \text{(C.9)}$$

In the sector of the three-linear objects constructed out three ψ and with one free Lorentz index, we just quote the structures which are SU(2) vectors entering in the computation of the Bianchi identities. They are:

$$\begin{aligned}
B_A^{1a} &= \gamma_7\psi_A\overline{\psi}_B\gamma^7\gamma^a\psi_C\epsilon^{BC}\,, \\
B_A^{2a} &= \gamma^7\gamma^a\psi_A\overline{\psi}_B\gamma_7\psi_C\epsilon^{BC}\,, \\
B_A^{3a} &= \gamma_b\psi_A\overline{\psi}_B\gamma^{ab}\psi_C\epsilon^{BC}\,, \\
B_A^{4a} &= \gamma^{ab}\psi_A\overline{\psi}_B\gamma_b\psi_C\epsilon^{BC}\,, \\
B_A^{5a} &= \gamma^7\gamma^{ab}\psi_A\overline{\psi}_B\gamma_7\gamma_b\psi_C\epsilon^{BC}\,, \\
B_A^{6a} &= \gamma^{abc}\psi_A\overline{\psi}_B\gamma_{bc}\psi_C\epsilon^{BC}\,, \\
B_A^{7a} &= \psi_A\overline{\psi}_B\gamma^a\psi_C\epsilon^{BC}\,.
\end{aligned}$$

Again, a group theoretical and numerical analysis give the following three relations among the seven quantities:

$$\begin{aligned}
B_A^{3a} &= -B_A^{1a} + \frac{1}{2}B_A^{5a} + \frac{1}{4}B_A^{6a} + \frac{3}{2}B_A^{7a}\,, \\
B_A^{4a} &= -B_A^{1a} - B_A^{5a} - B_A^{7a}\,, \\
B_A^{2a} &= B_A^{1a} + \frac{1}{2}B_A^{5a} - \frac{1}{4}B_A^{6a} - \frac{1}{2}B_A^{7a}\,.
\end{aligned} \tag{C.10}$$

The quoted relations (C.3)–(C.8) and (C.10) are all we need for the solution of Bianchi identities and the construction of the lagrangian.

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